

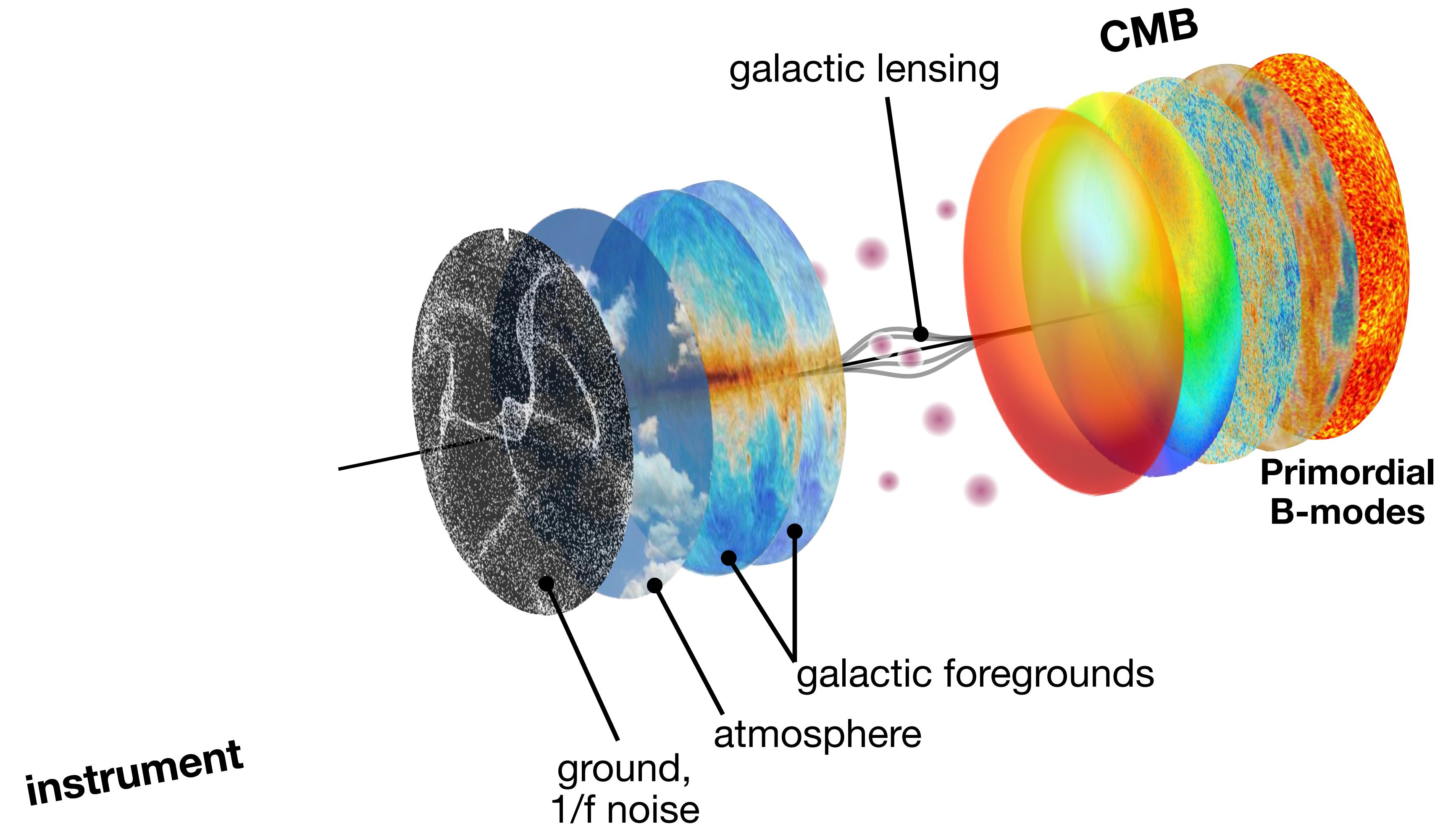
Towards a parametric model to describe HWP non-idealities for primordial B-modes detection

CMB-CAL @ Bicocca, Milan 2024

Ema Tsang King Sang, PhD student @ APC Paris, under the supervision of J.Errard & R.Stompor



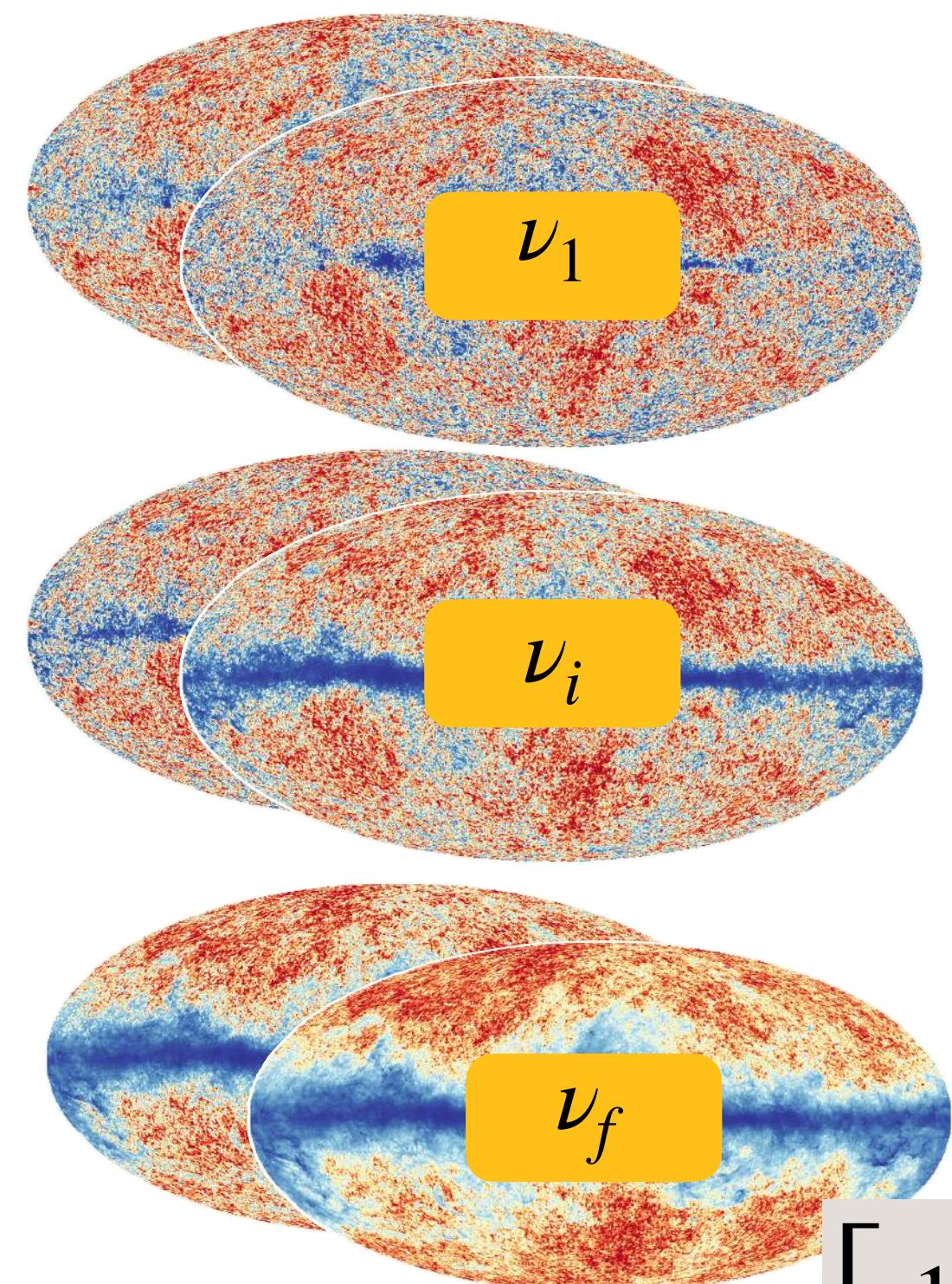
Observational challenges



Credit: Josquin Errard

A short recap of component separation

Frequency maps:
Q/U maps for each observed frequency

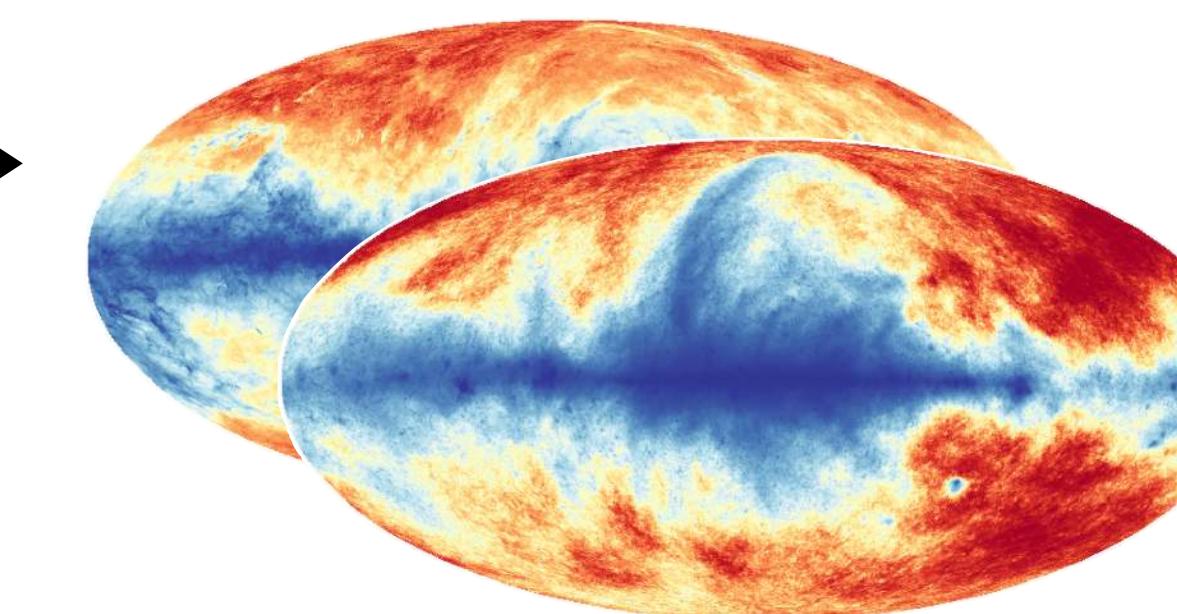
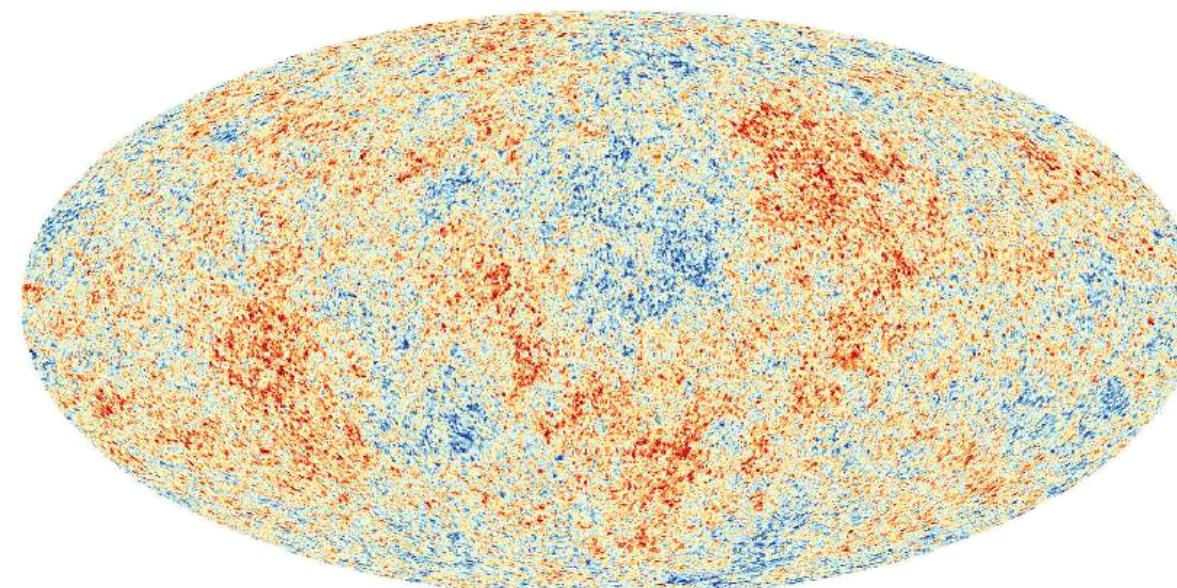


Data model (observed):



$$d = \begin{bmatrix} 1 & a^{dust}(\nu_1) & a^{synch}(\nu_1) \\ \vdots & \vdots & \vdots \\ 1 & a^{dust}(\nu_f) & a^{synch}(\nu_f) \end{bmatrix}$$

$$\begin{bmatrix} s_{cmb} \\ s_{dust} \\ s_{synch} \end{bmatrix} + n$$



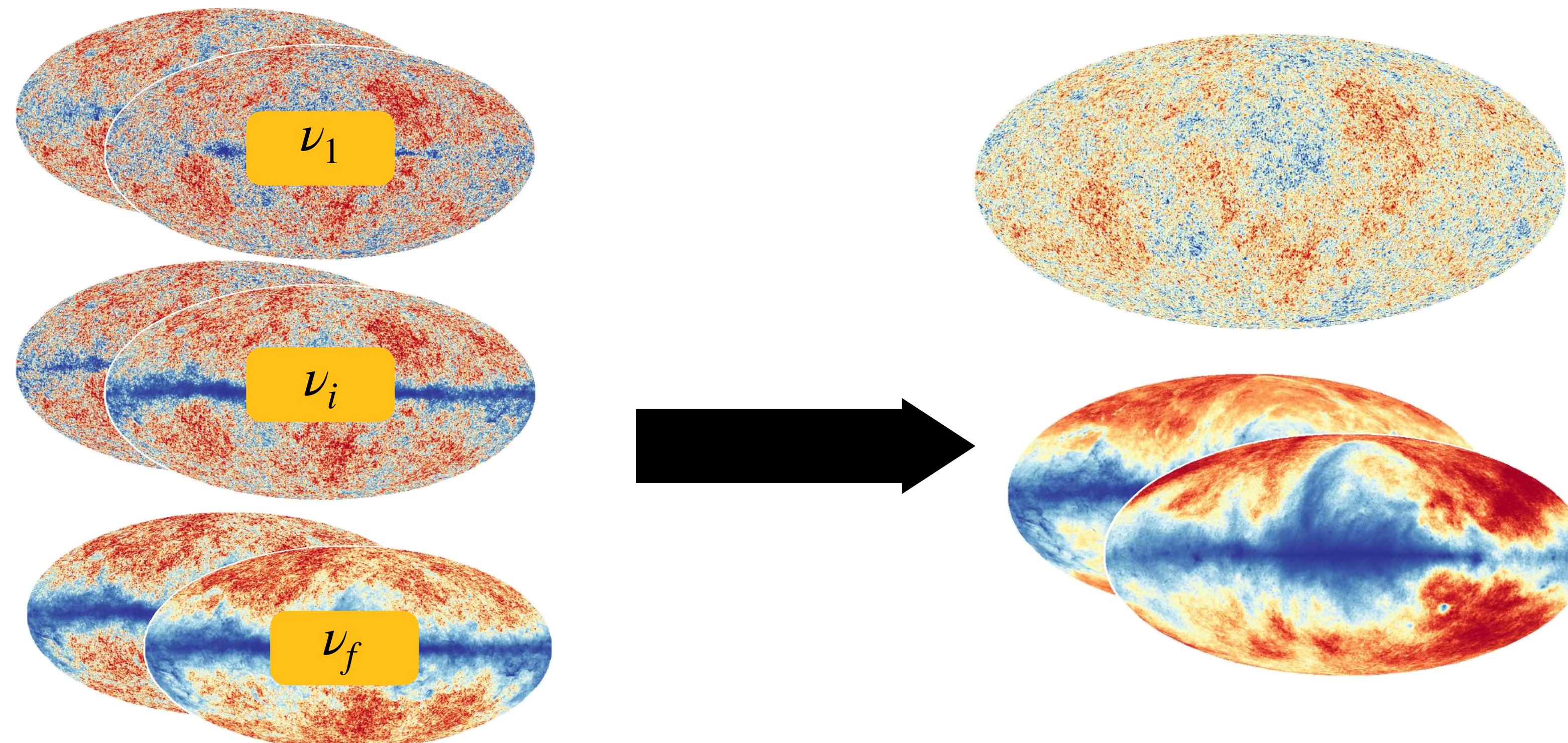
Component maps:
Q/U map for each component

$A(\beta, \nu)$ mixing matrix (Nfreq x Ncomponent):

gives the frequency scaling of each sky component (MBB for dust, power law for synchrotron),
assumed parametrised by *spectral parameters β*

A short recap of component separation

Frequency maps:
Q/U maps for each observed frequency

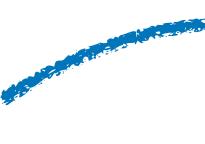


Component maps:
Q/U map for each component

Data model (observed): $d = \mathbf{A}(\beta) s + n$

generalised least squares solution

Likelihood of the data: $2 \log \mathcal{L}(s, \beta) = (d - \mathbf{A}s)^T \mathbf{N}^{-1} (d - \mathbf{A}s)$  $\tilde{s} = (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} d$

Spectral likelihood: $2 \log \mathcal{L}_{spec}(\beta) = d^T \mathbf{N}^{-1} \mathbf{A} (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} d$  $\tilde{\beta}$

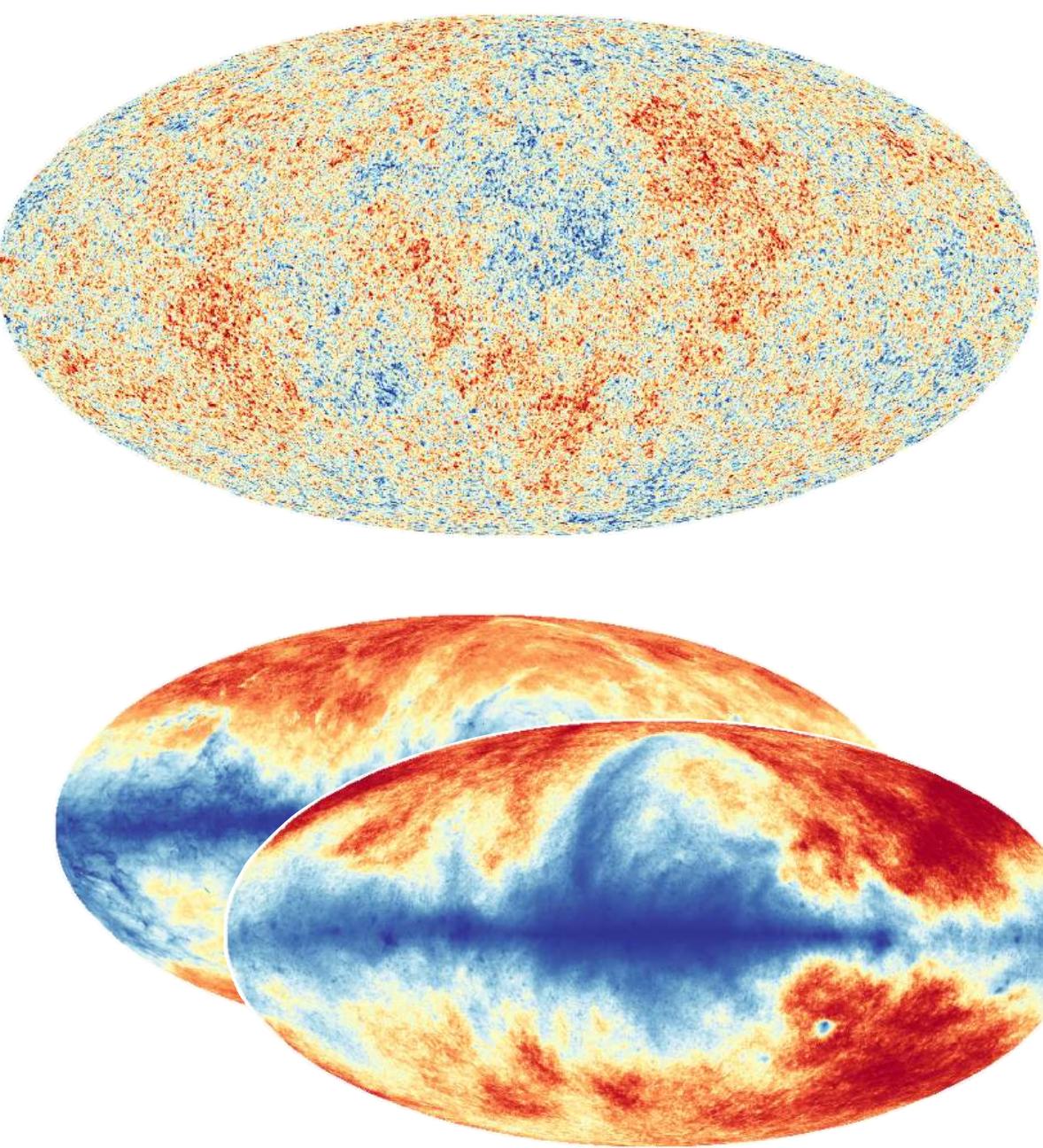
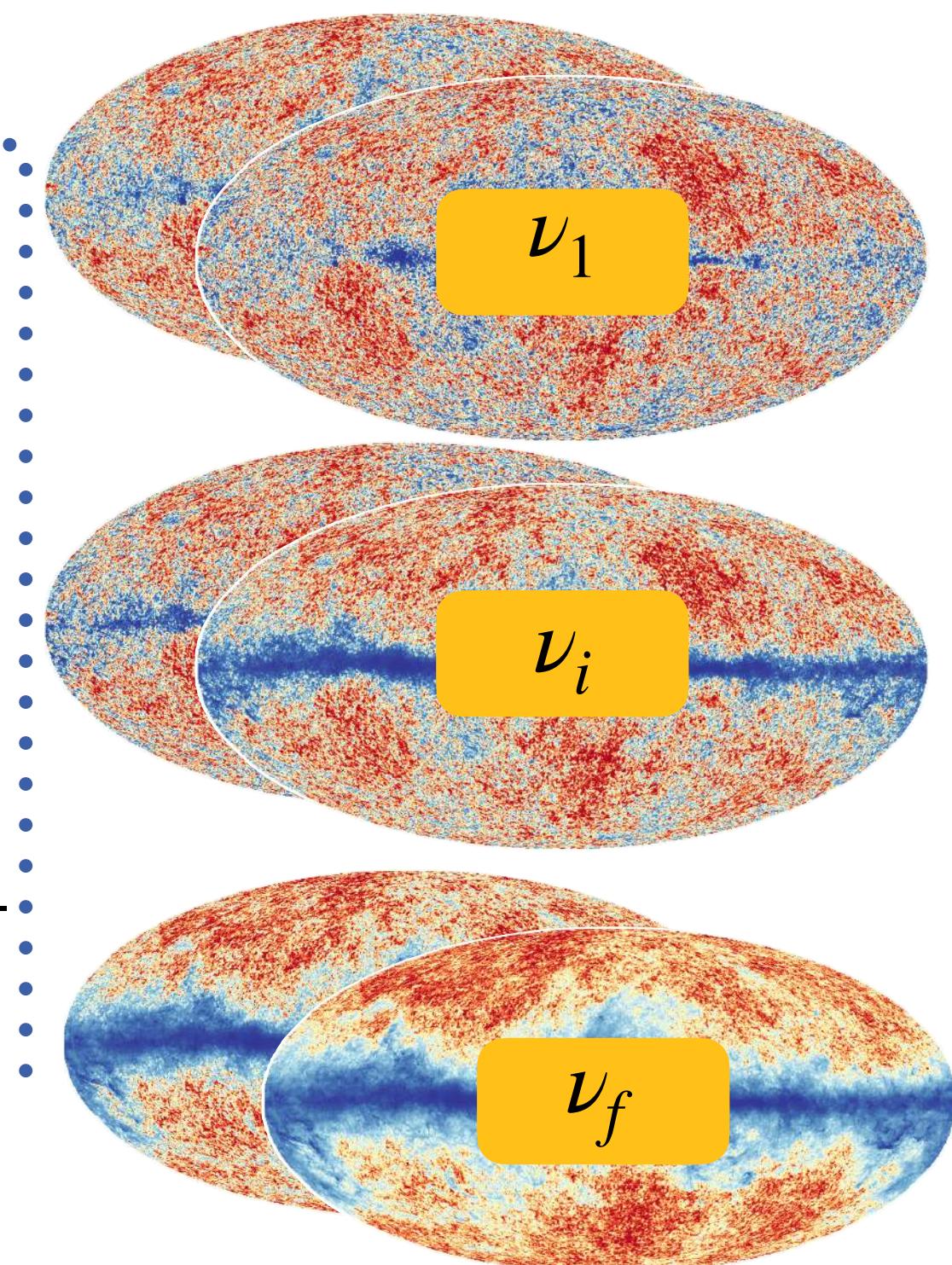
Stompor & al. (2009)

Generalised component separation

Vergès et al. (2020)
Jost et al. (2023)

- Frequency maps:
 - Q/U maps
 - integrated maps
 - for each observed frequency

$$\bar{\mathbf{X}}(\nu_c) = \frac{\int d\nu B(\nu, n\nu_c) \mathbf{X}(\nu)}{\int d\nu B(\nu)}$$



- Component maps:
 - Q/U map for each component

Data model (observed):

$$d = \int d\nu B(\nu) \mathbf{M}_{inst}(\gamma, \nu) \mathbf{A}(\beta, \nu) s + n$$

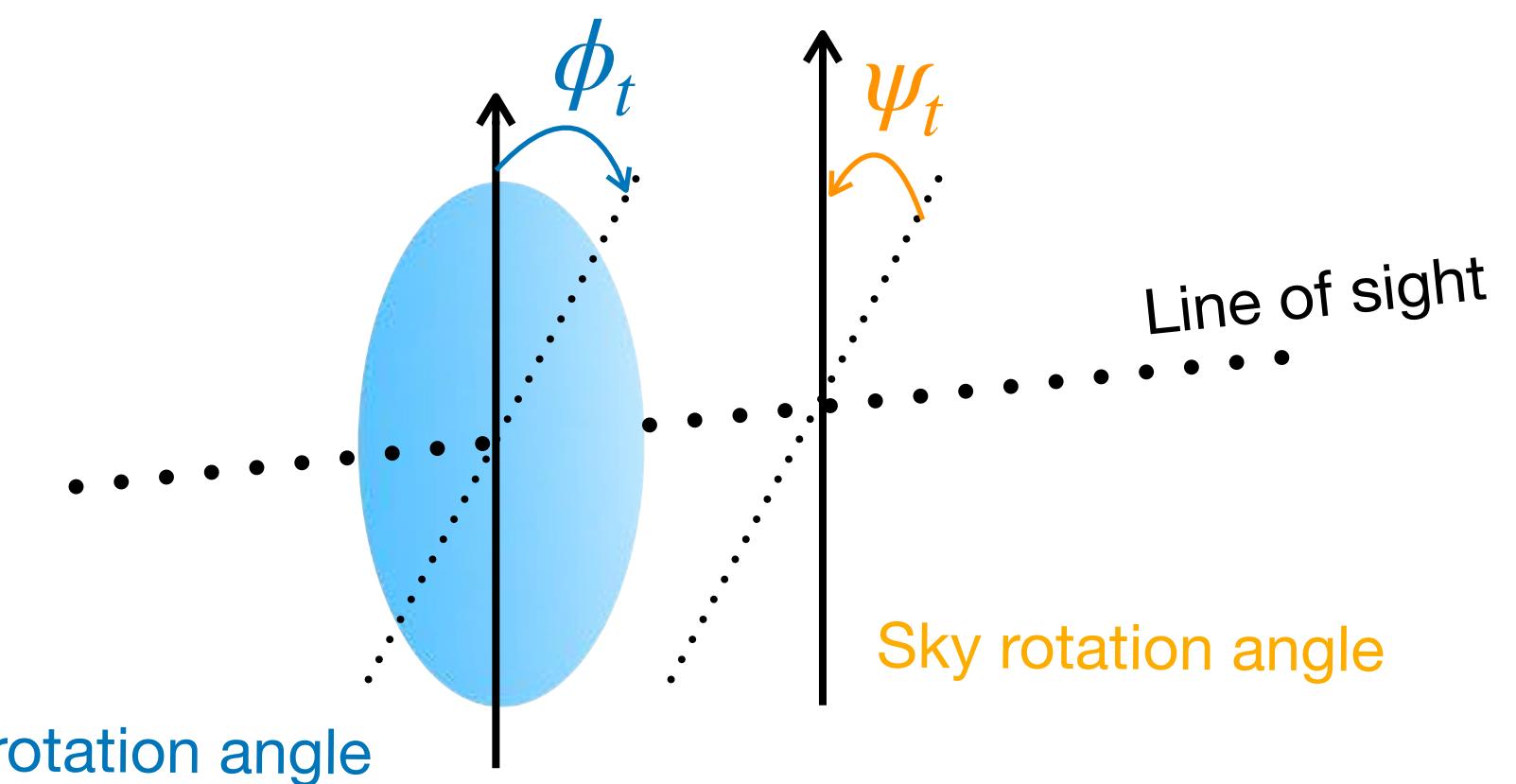
Generalised mixing matrix:
parametrised by *spectral parameters* and *hardware parameters*
→ HWP effects

Generalised component separation

Vergès et al. (2020)

Monochromatic model at the map level:

$$\begin{aligned} d &= I + \cos(4\phi + 2\psi_t) Q + \sin(4\phi + 2\psi_t) U \\ &= M_{00}I + M_{01}Q + M_{02}U \end{aligned}$$



Generalised component separation

Vergès et al. (2020)

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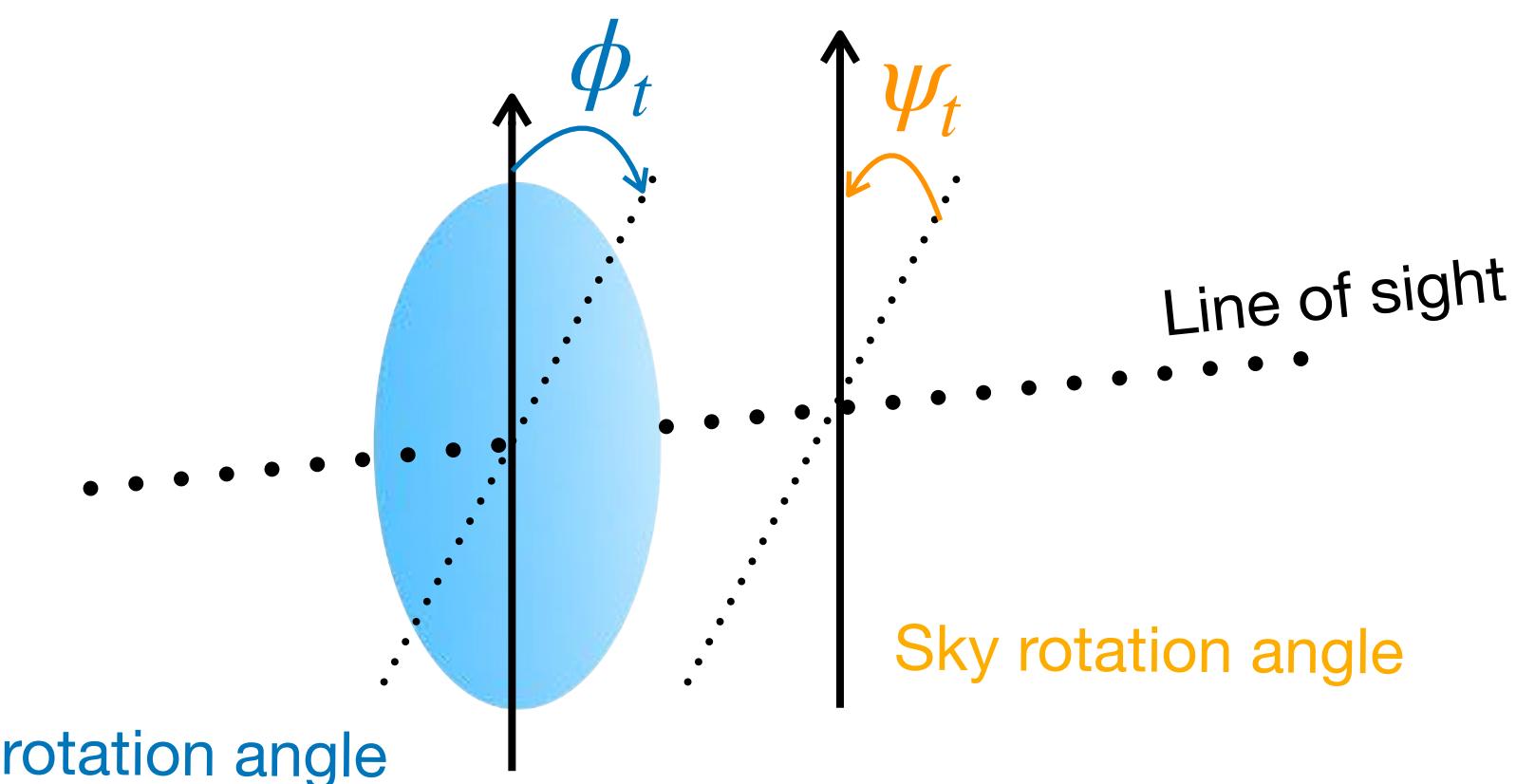
Multifrequency model:

$$M_{0i} = \sum_{k=0,4} C_{i,k} \cos(k\phi + 2\psi_t) + \sum_{k=0,4} S_{i,k} \sin(k\phi + 2\psi_t)$$

regroup terms according to their time dependence: **mixed Stokes maps**

$$\begin{pmatrix} \tilde{I}(\gamma_t, \nu) \\ \tilde{C}_0(\gamma_t, \nu) \\ \tilde{S}_0(\gamma_t, \nu) \\ \tilde{C}_4(\gamma_t, \nu) \\ \tilde{S}_4(\gamma_t, \nu) \end{pmatrix} = \begin{bmatrix} M_{00}(\nu) & 0 & 0 \\ 0 & C_{Q;0}(\nu) & C_{U;0}(\nu) \\ 0 & S_{Q;0}(\nu) & S_{U;0}(\nu) \\ 0 & C_{Q;4}(\nu) & C_{U;4}(\nu) \\ 0 & S_{Q;4}(\nu) & S_{U;4}(\nu) \end{bmatrix} \mathbf{A}(\beta, \nu) \begin{pmatrix} I(\gamma_t, \nu) \\ Q(\gamma_t, \nu) \\ U(\gamma_t, \nu) \end{pmatrix}$$

$$d = \mathbf{M}_{hwp}(\gamma, \nu) \mathbf{A}(\beta, \nu) s$$



Generalised component separation

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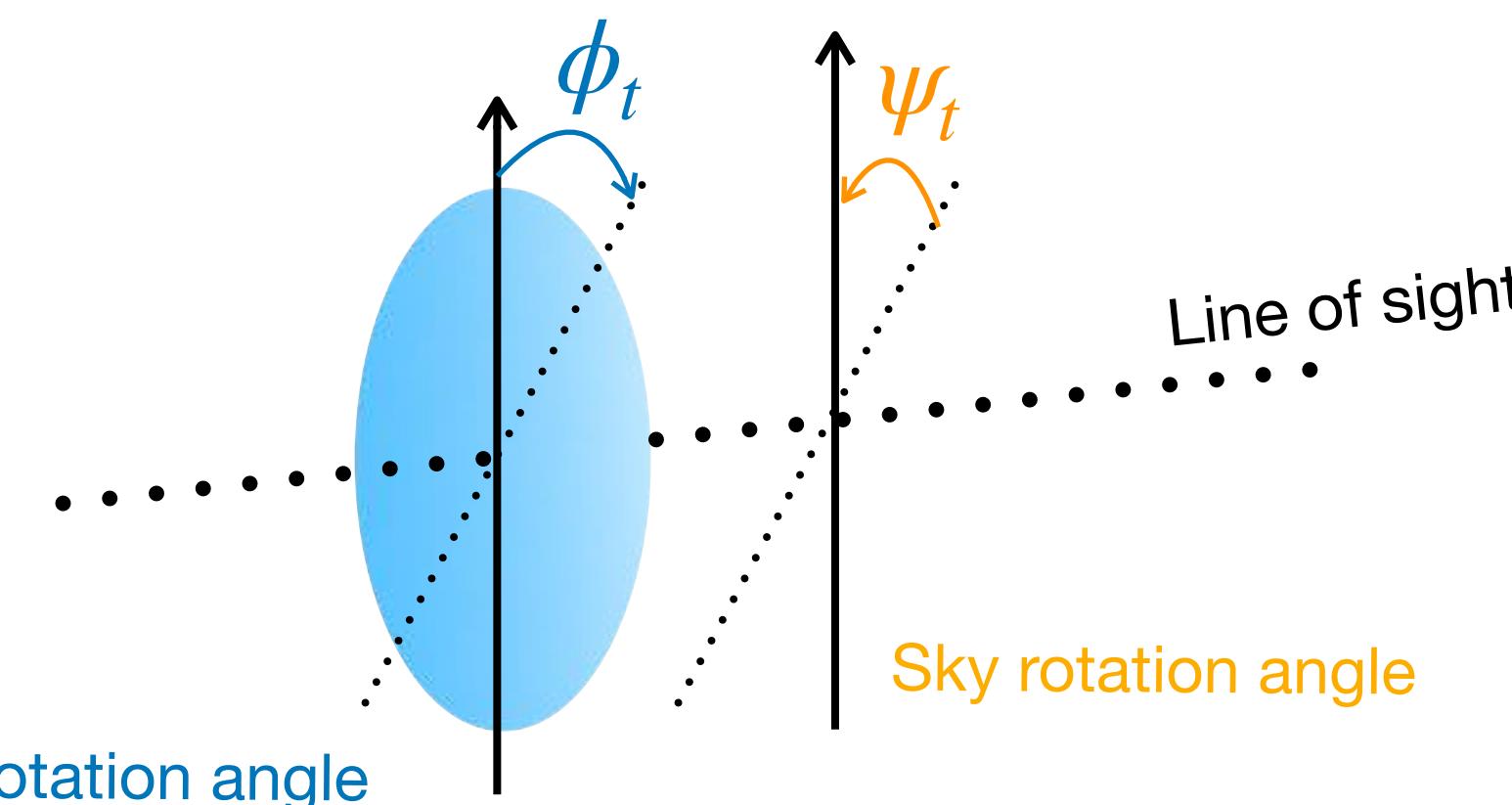
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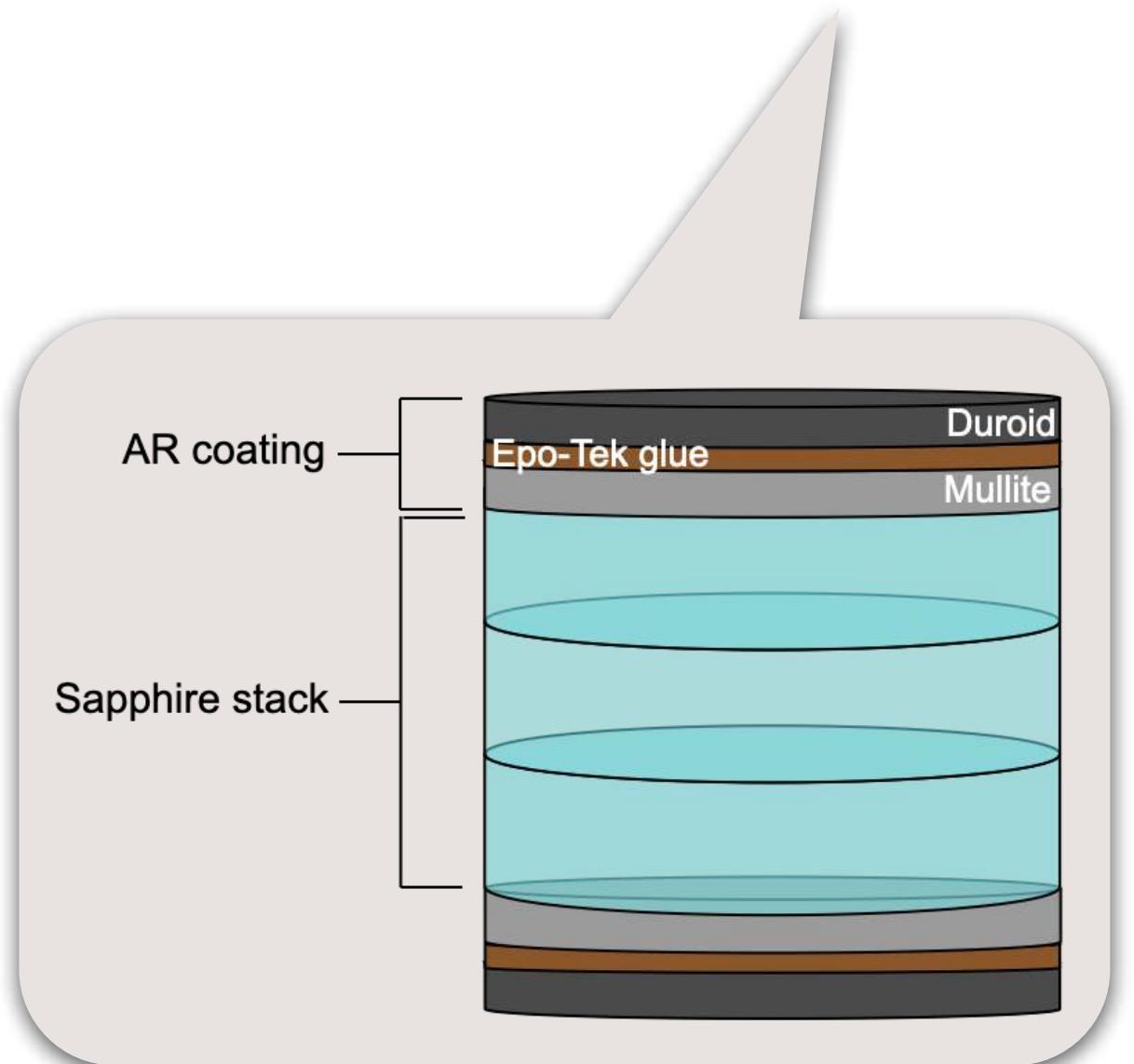


- Output of the mapmaking procedure are not pure Q/U maps at each frequency but **mixed Stokes maps**
 - under the assumption of a perfectly uniform sky coverage
 - not trivial Q/U demodulation
- **Estimation of HWP** parameters from component separation:
 - **need of priors**, necessity hardware calibration
 - considering the fact that they apply to more than one frequency band
- Interplay between instrumental and foreground parameters
 - mitigation: **time domain component separation**

Extension of the former model

Increased sensitivity of CMB experiment requires control of systematics:
need for a reliable model of the optic chain

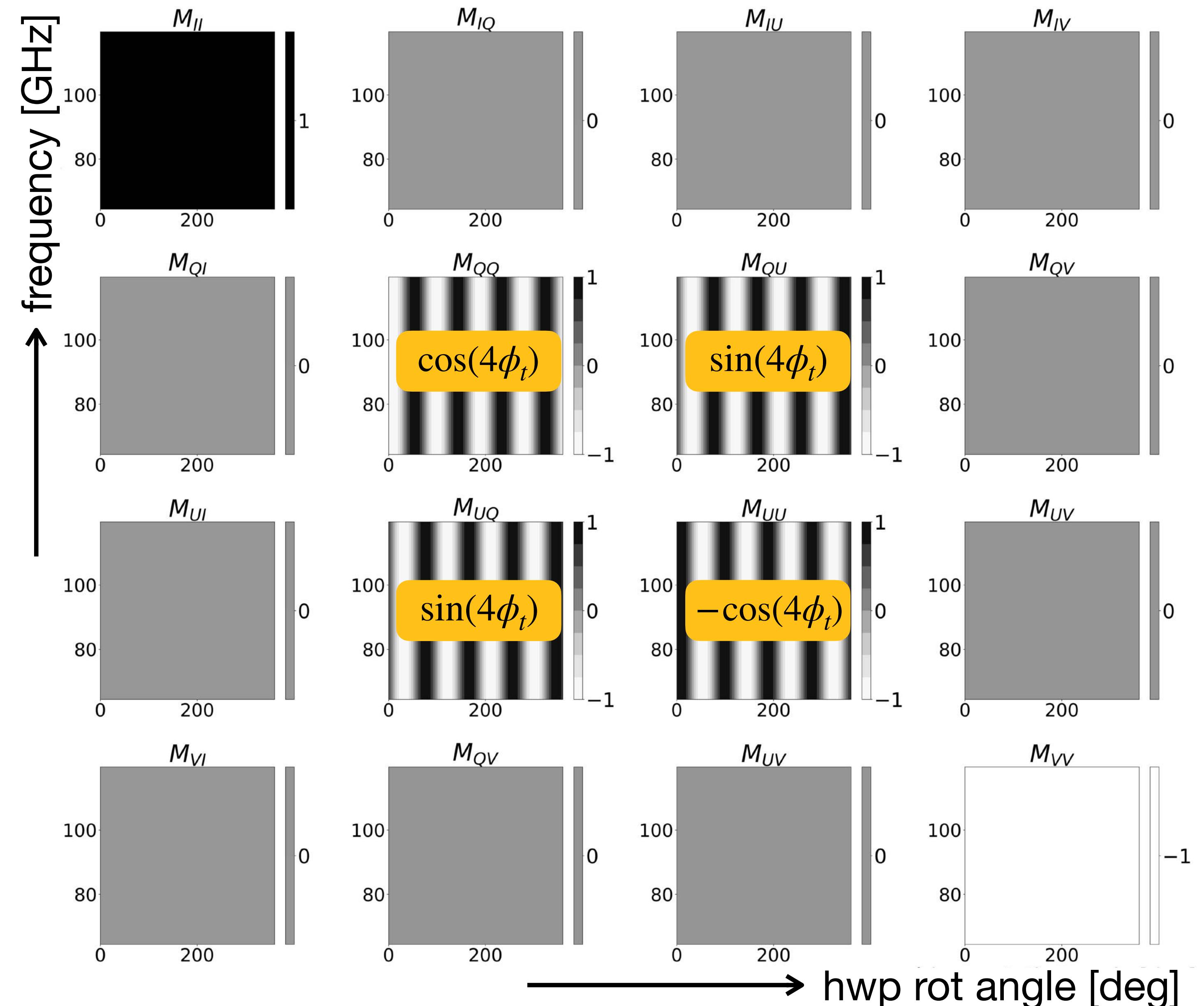
- Consider a **frequency-dependent analytical model** for the hwp and study
- 2 cases:
 - ▶ **No bandpass case:**
→ An effective hwp can be built to make up for non-idealities of the hwp
 - ▶ **Band-pass integrated case:**
→ No effective model efficient enough to reduce the bias due to hwp-related systematics



Extension of the former model

Monochromatic model:

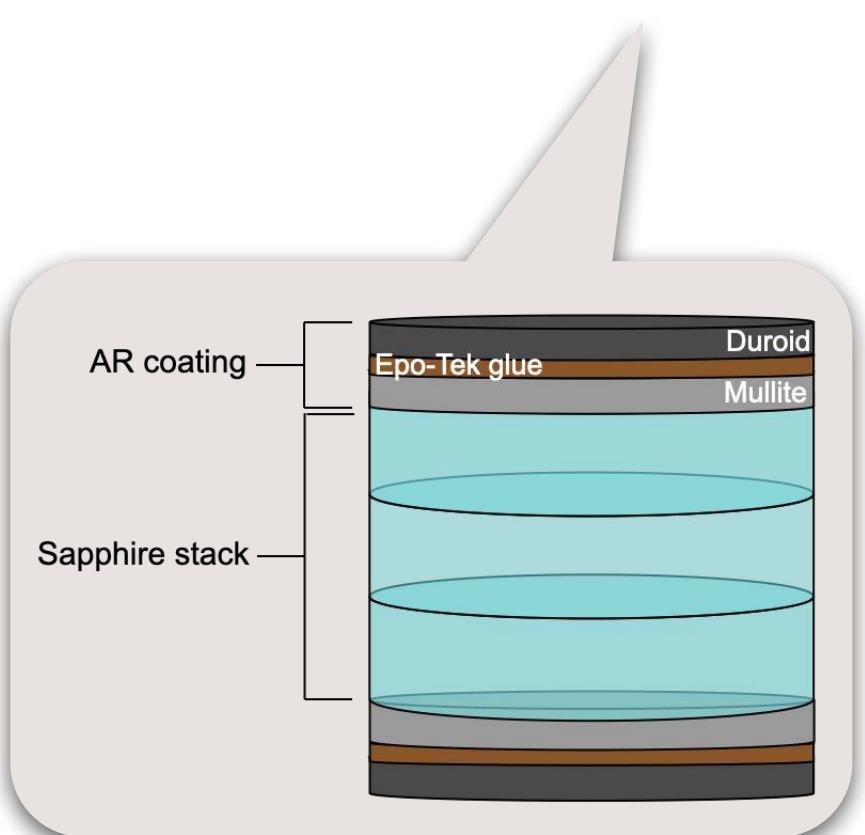
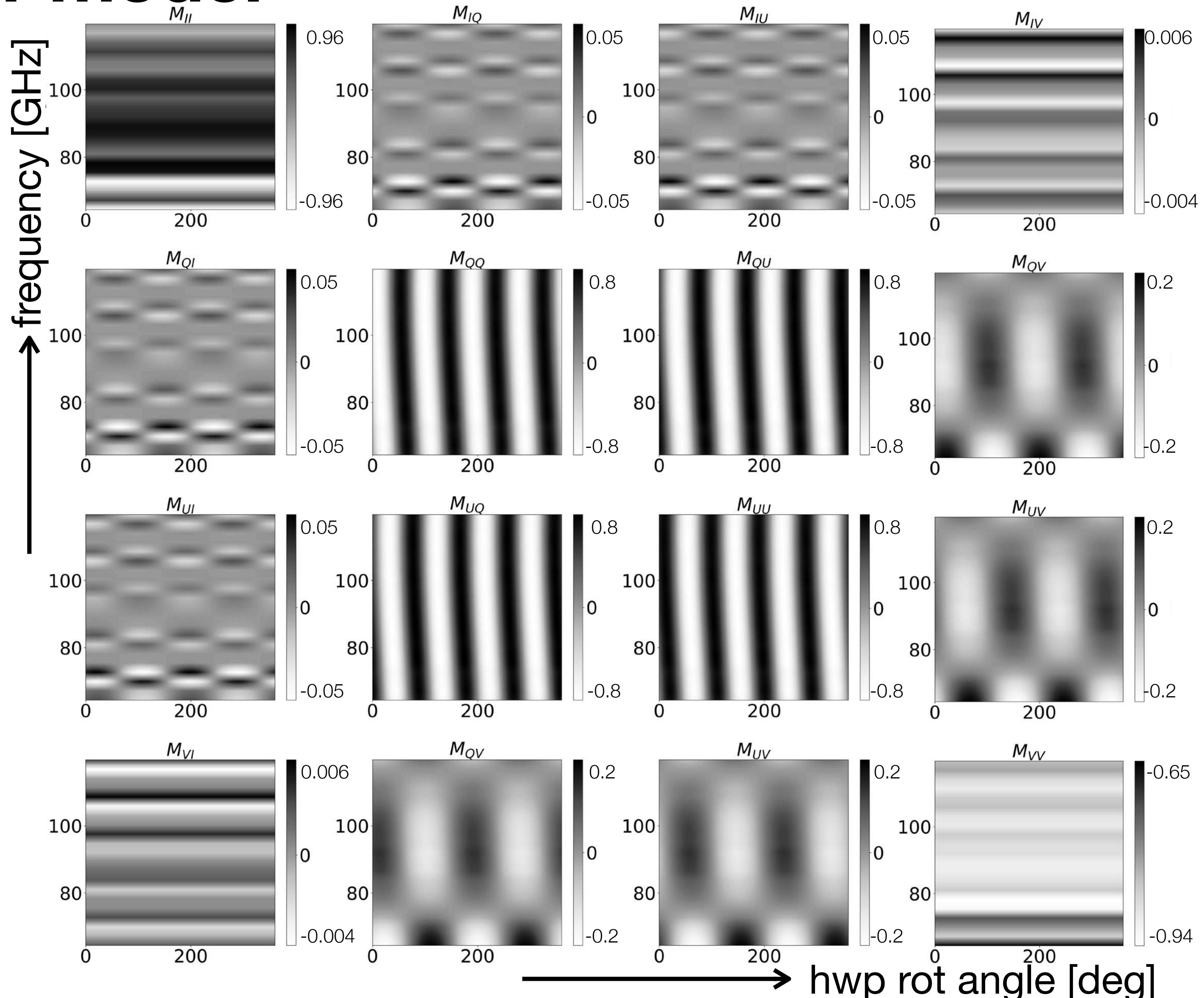
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Extension of the former model

Parameters from Sakaguri & al. (2024)
MF2, 0° incident angle

- Effects of transmission and reflection occurring in a hwp stack: transfer matrix method from Essingler-Hileman (2014)

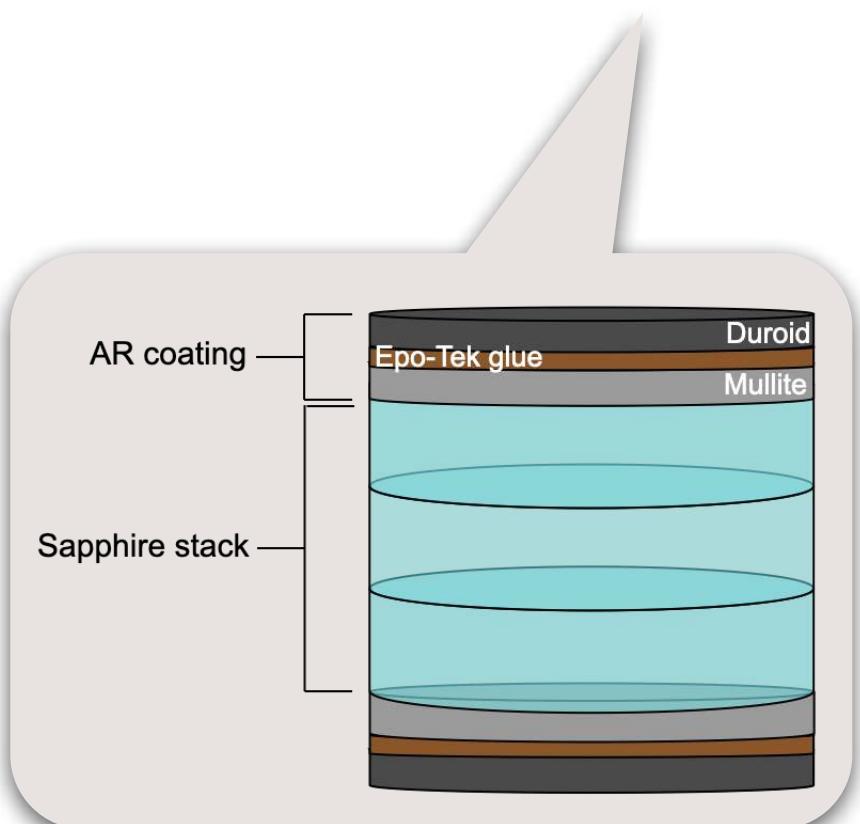
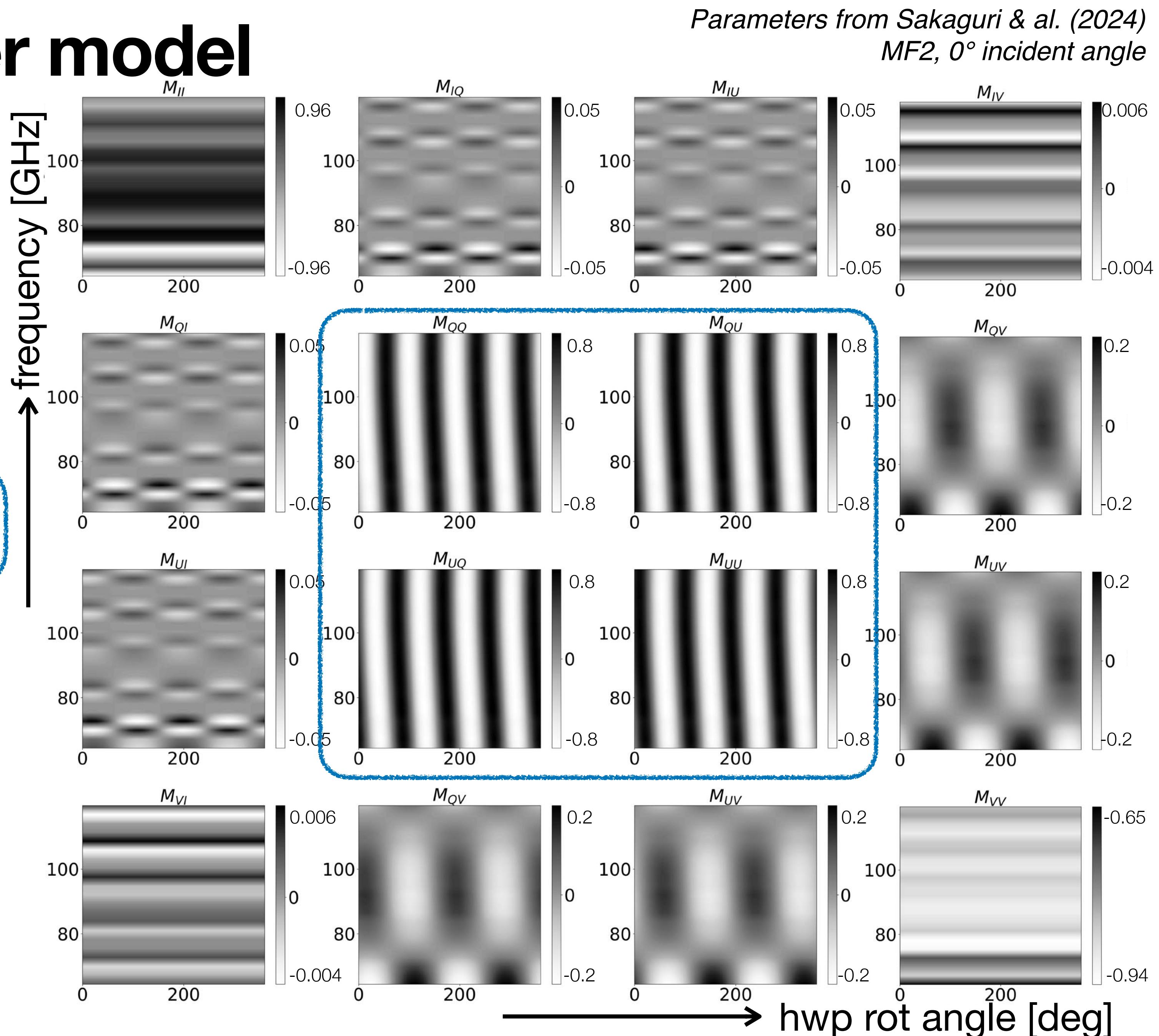


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- Effects of transmission and reflection occurring in a hwp stack: transfer matrix method from *Essinger-Hileman (2014)*

*Frequency dependent phase-shift
*Less efficient modulation than the ideal case



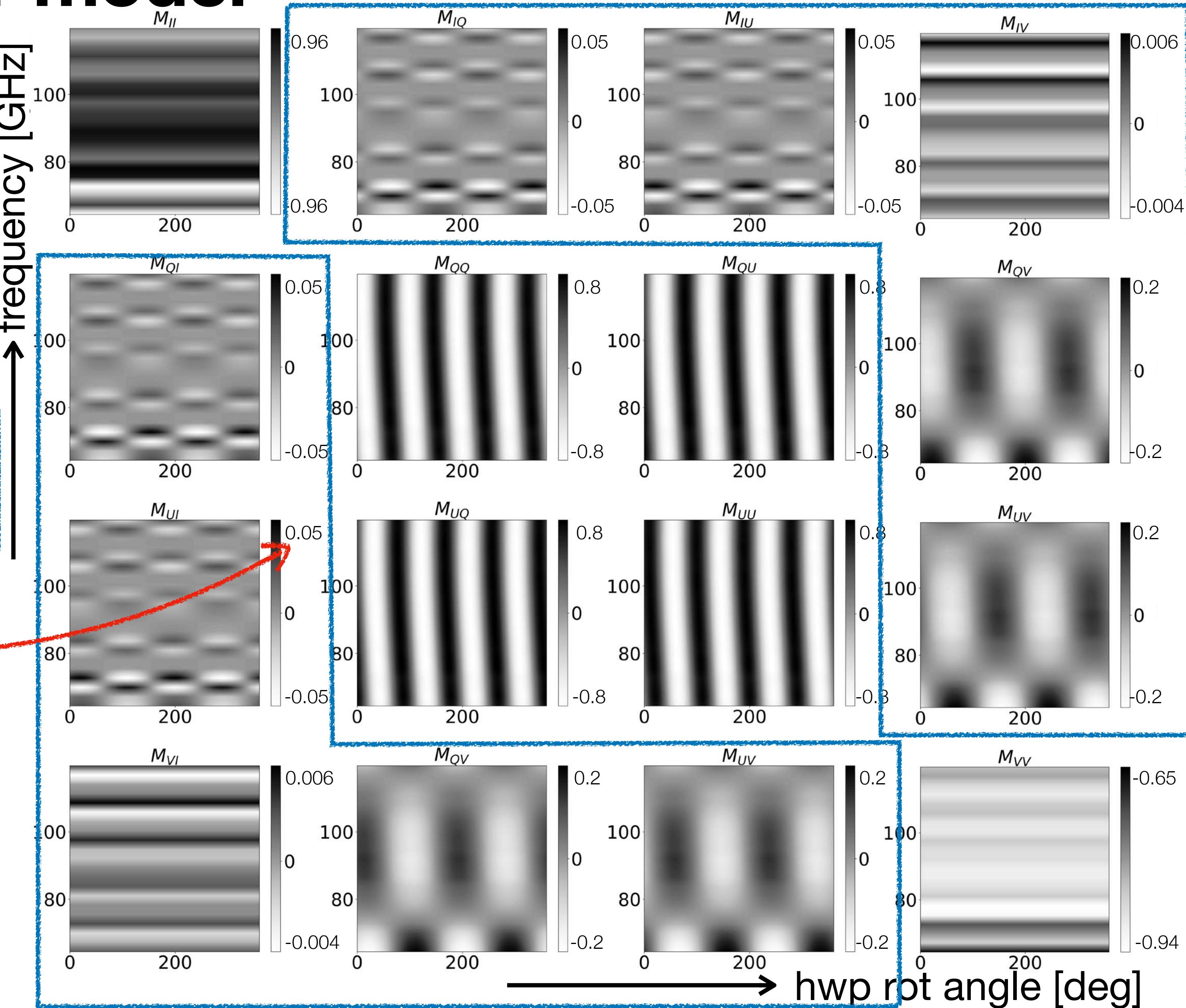
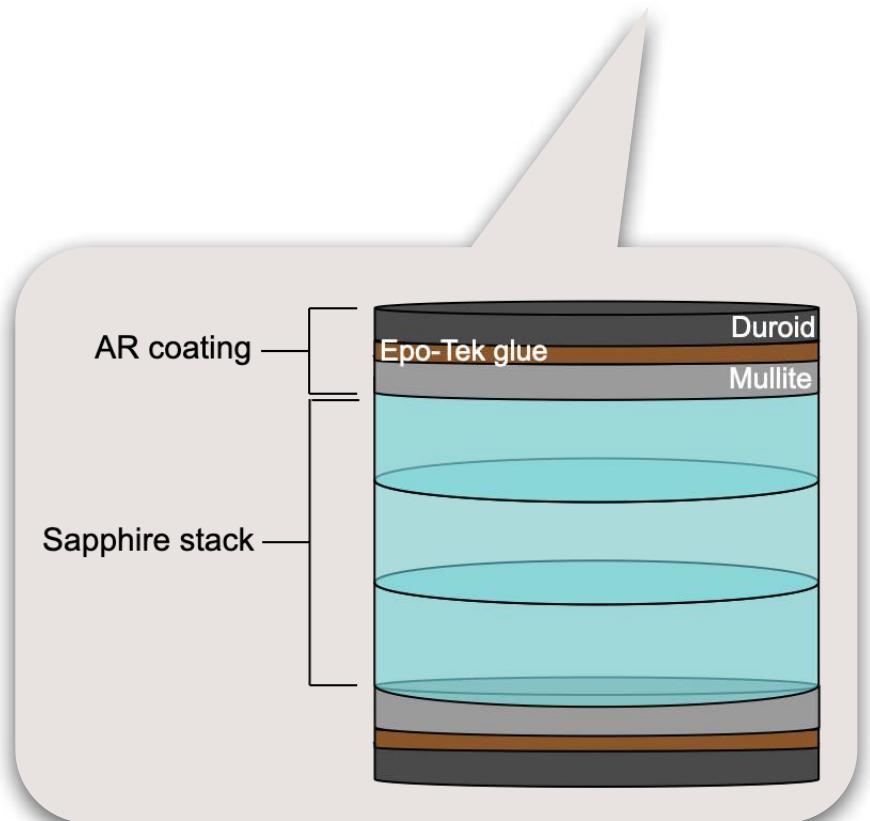
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- * $I \rightarrow P$ leakage + $V \rightarrow Q/U$
- * 2f modulation



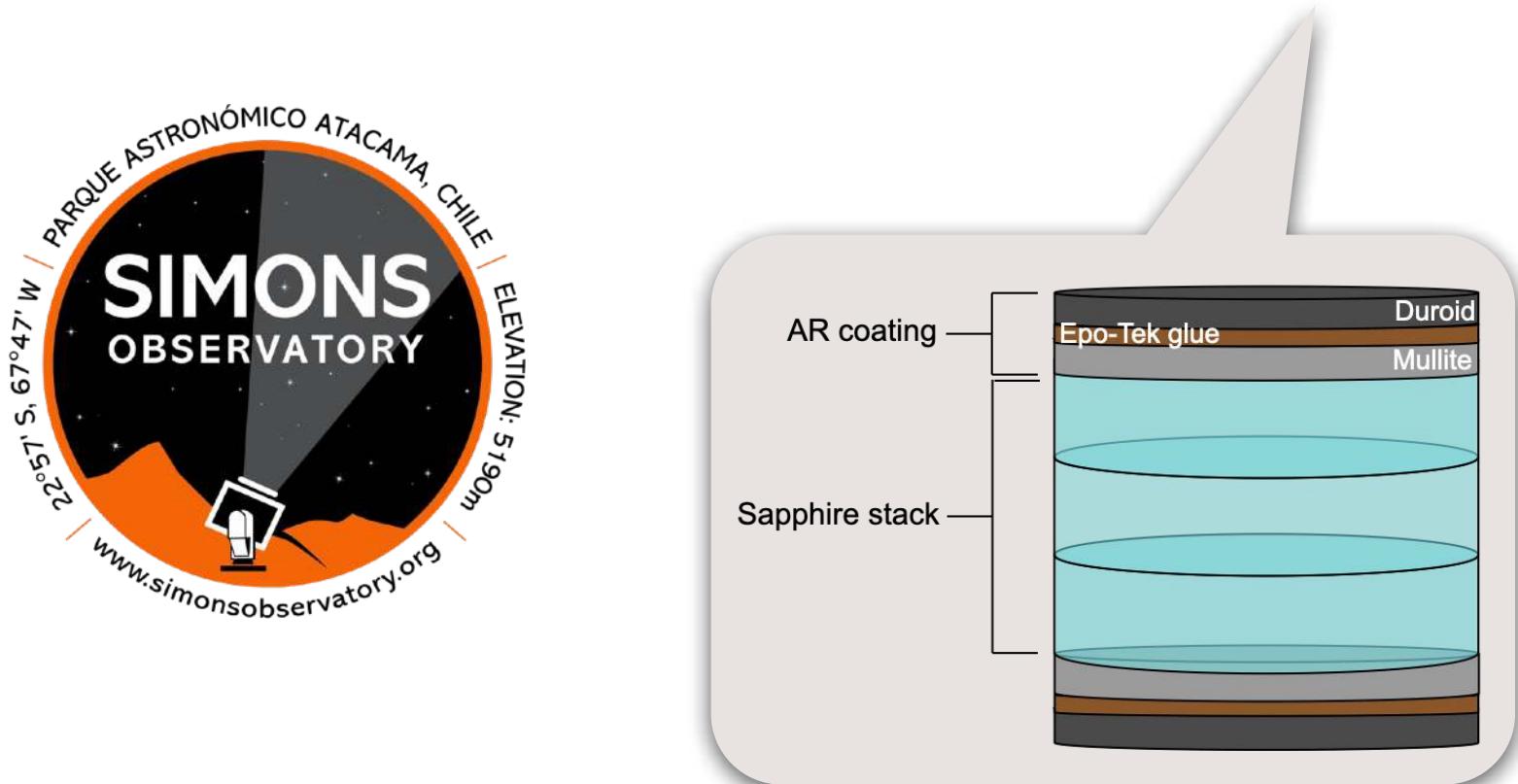
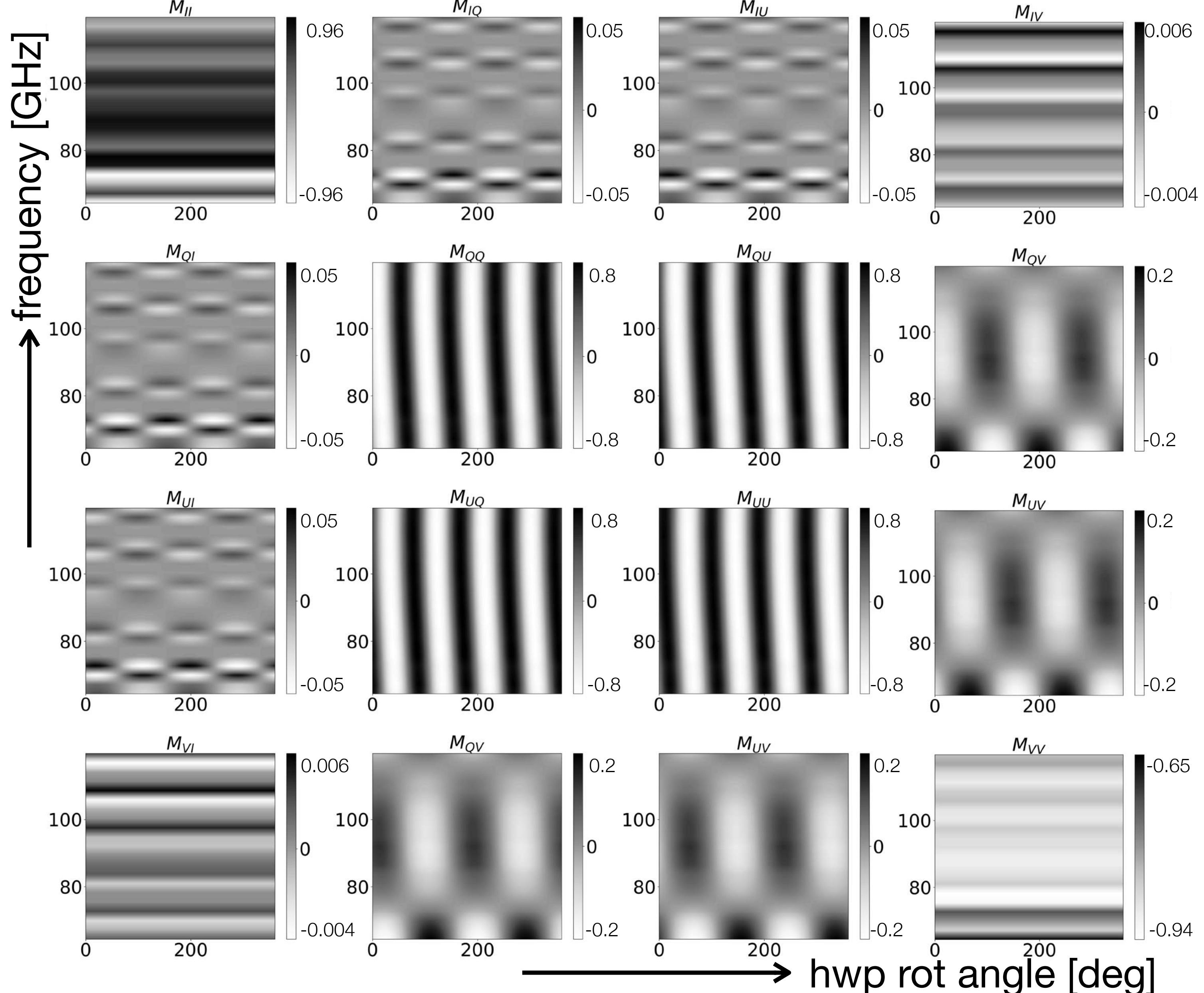
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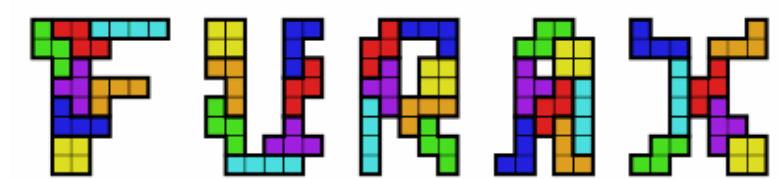
- *Frequency dependent phase-shift
- *Less efficient modulation than the ideal case
- * $I \rightarrow P$ leakage + $V \rightarrow Q/U$
- *2f modulation
- *Relevant parameters: layers thickness, incident angle (for each det)

This figure is derived from EM theory, so is expected even for a perfectly known hwp



No band-pass case

Pipeline:



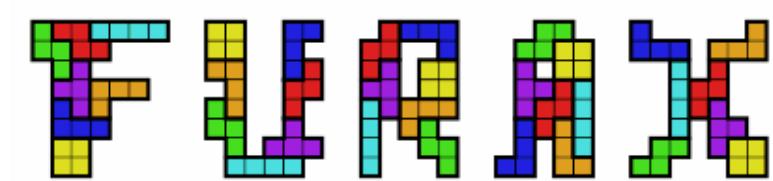
→ See Josquin's talk on Friday!

- ▶ **Modularity, extensibility, simplicity:**

```
h = pol @ hwp @ mixing_matrix @ sampling  
sol = ((h.T @ h).I @ h.T)(tod)
```

No band pass case

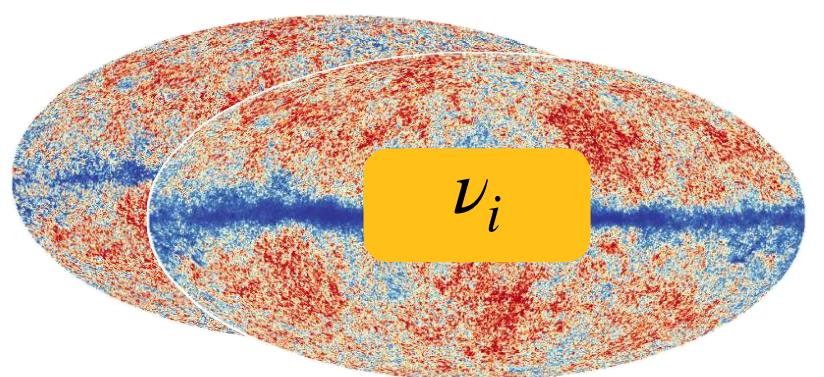
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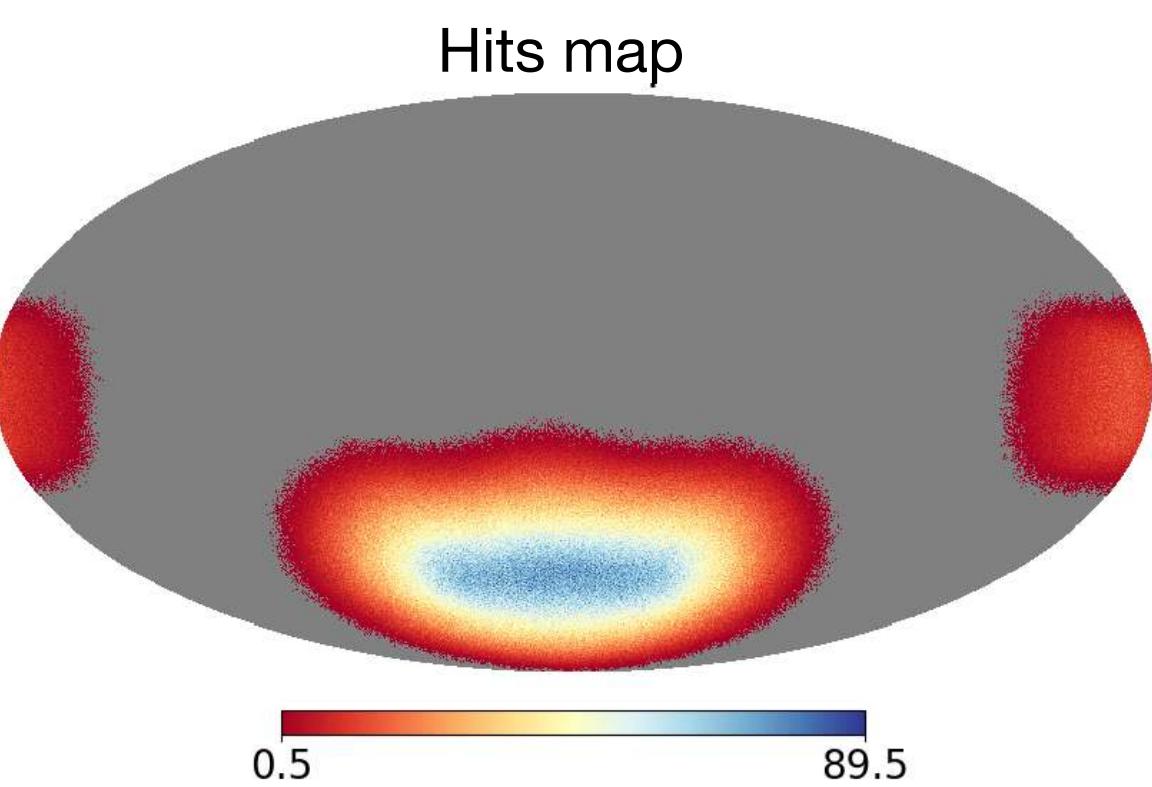
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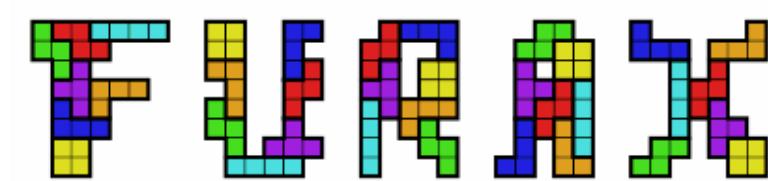
Q/U single frequency maps

- PySm templates:
cmb, dust @ 353 GHz,
synchrotron @ 30 GHz
- No noise
- Scanning strategy:
random hits on SO
SAT like mask



No band pass case

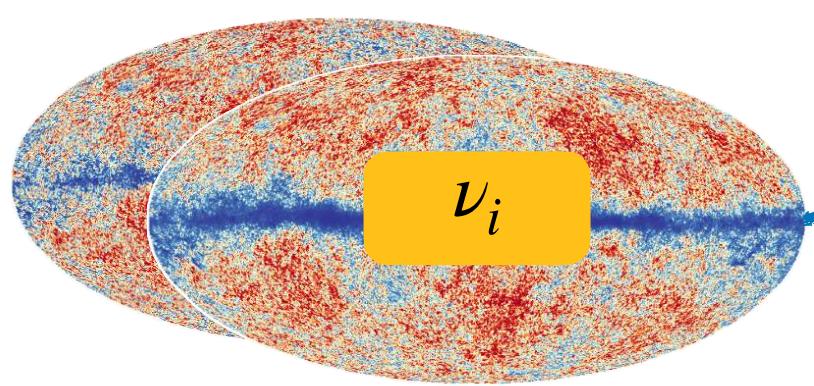
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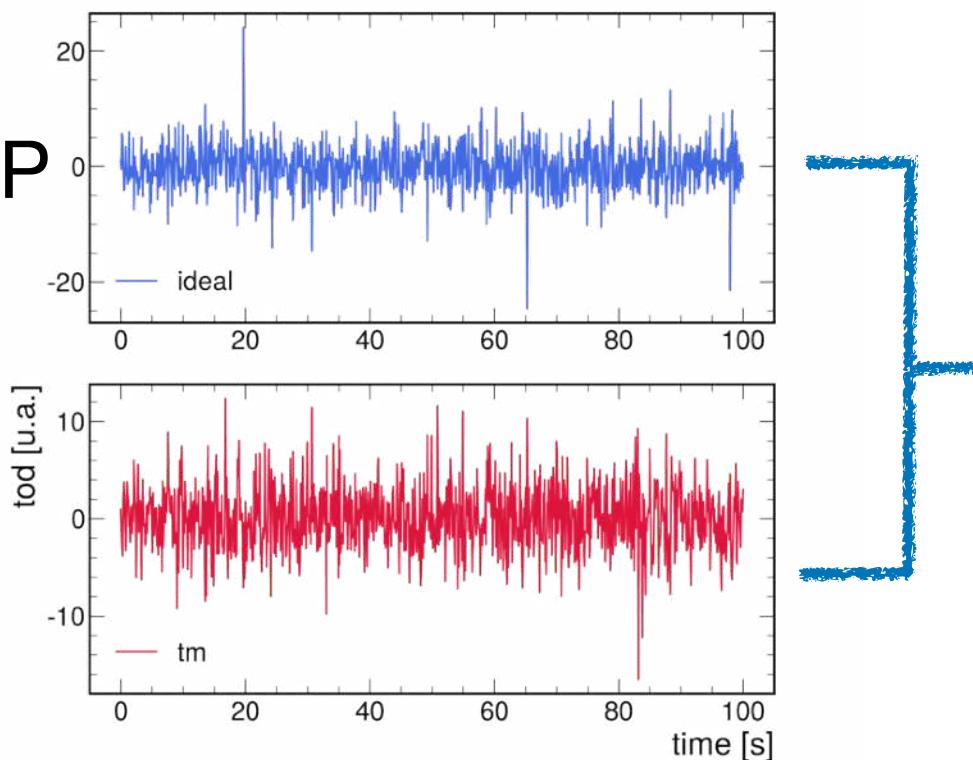
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HWP modulation

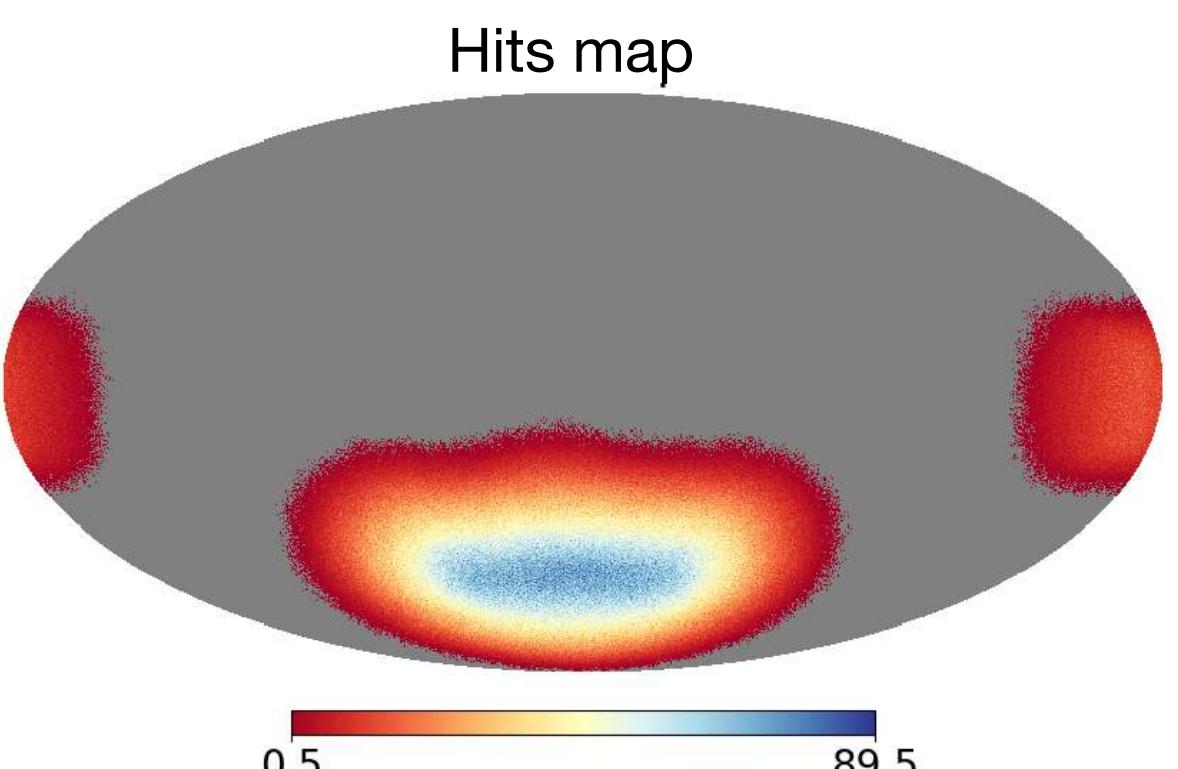
Ideal HWP
Non ideal HWP



Ideal HWP sky reprojection

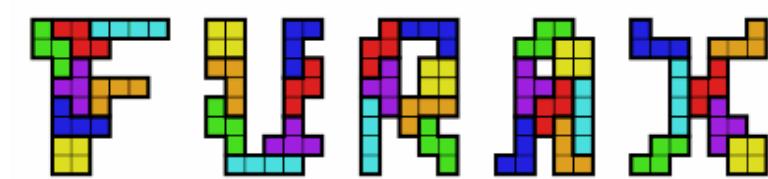
Q/U residuals single frequency maps

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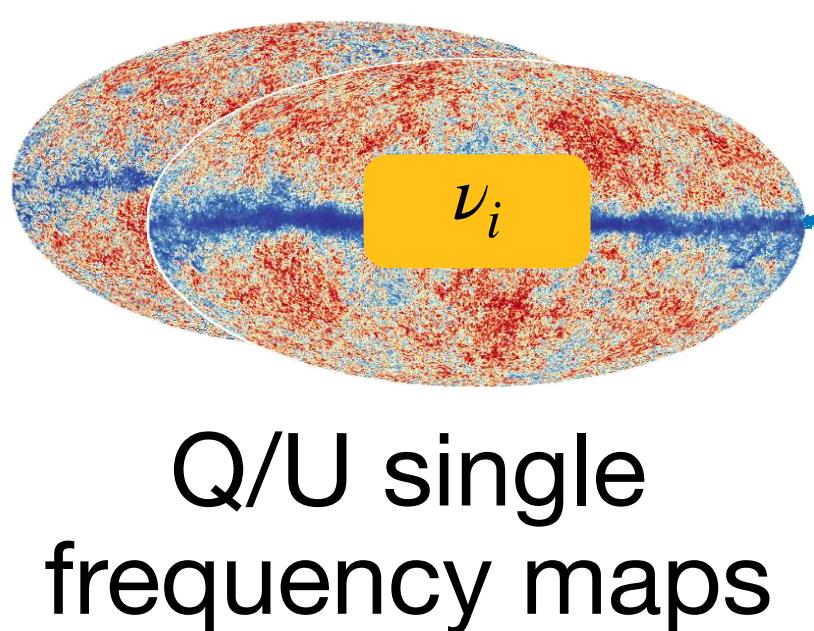
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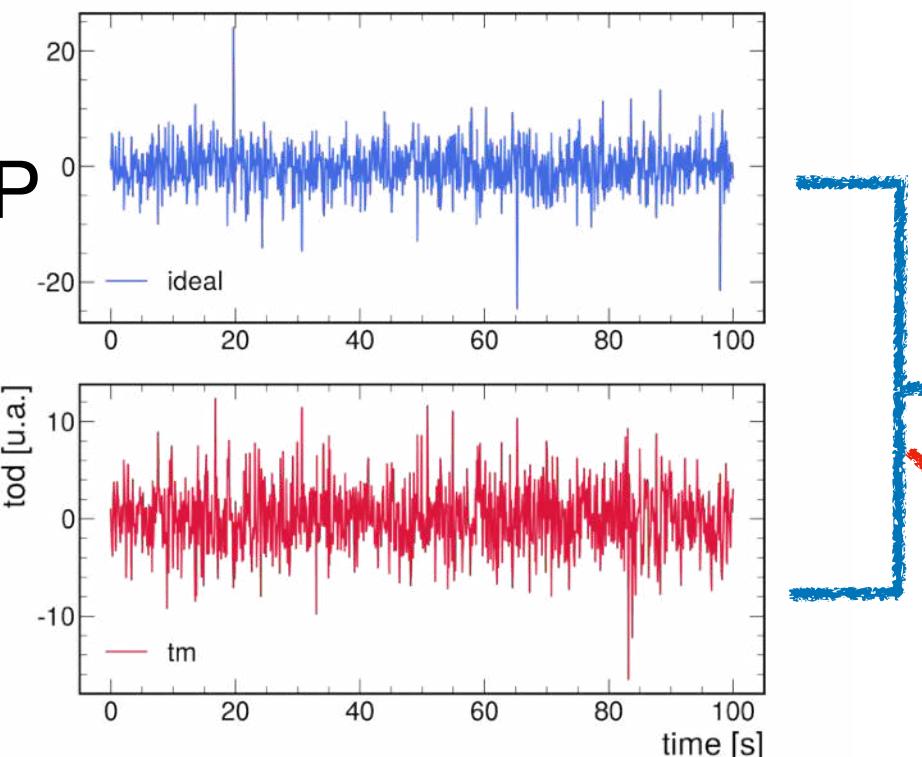
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HWP modulation

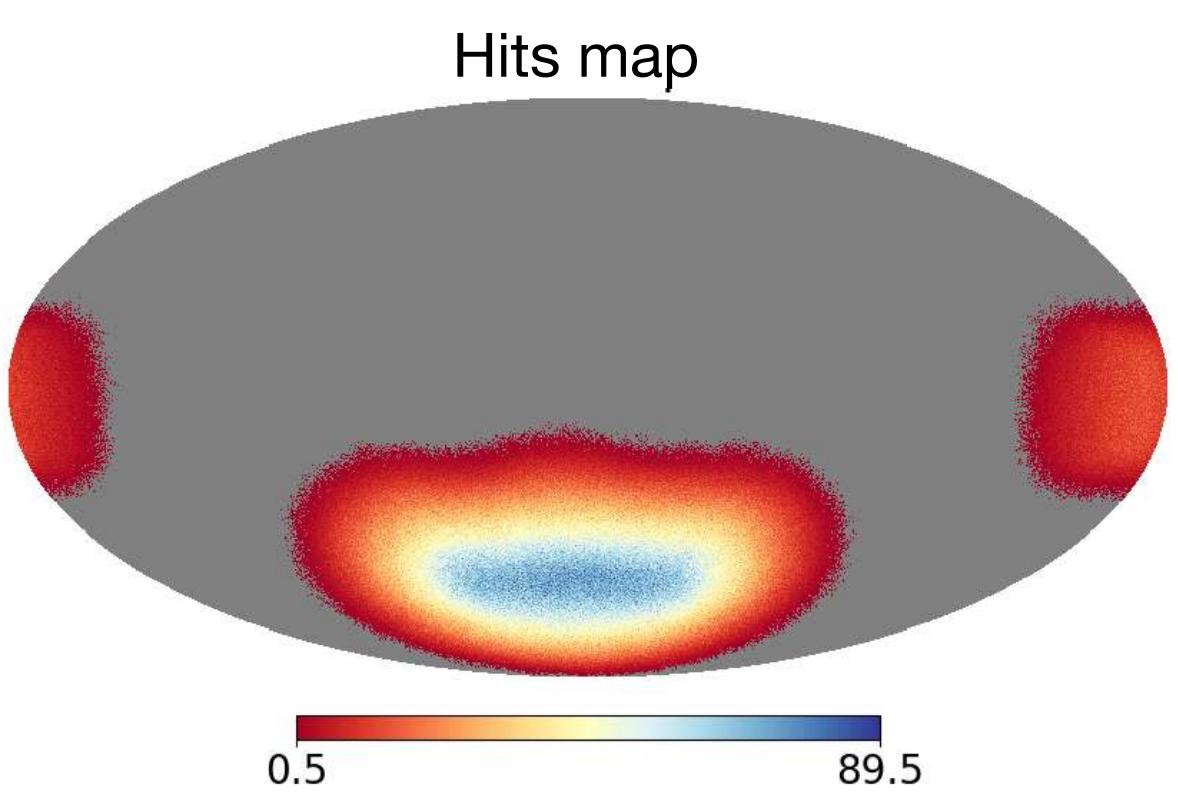
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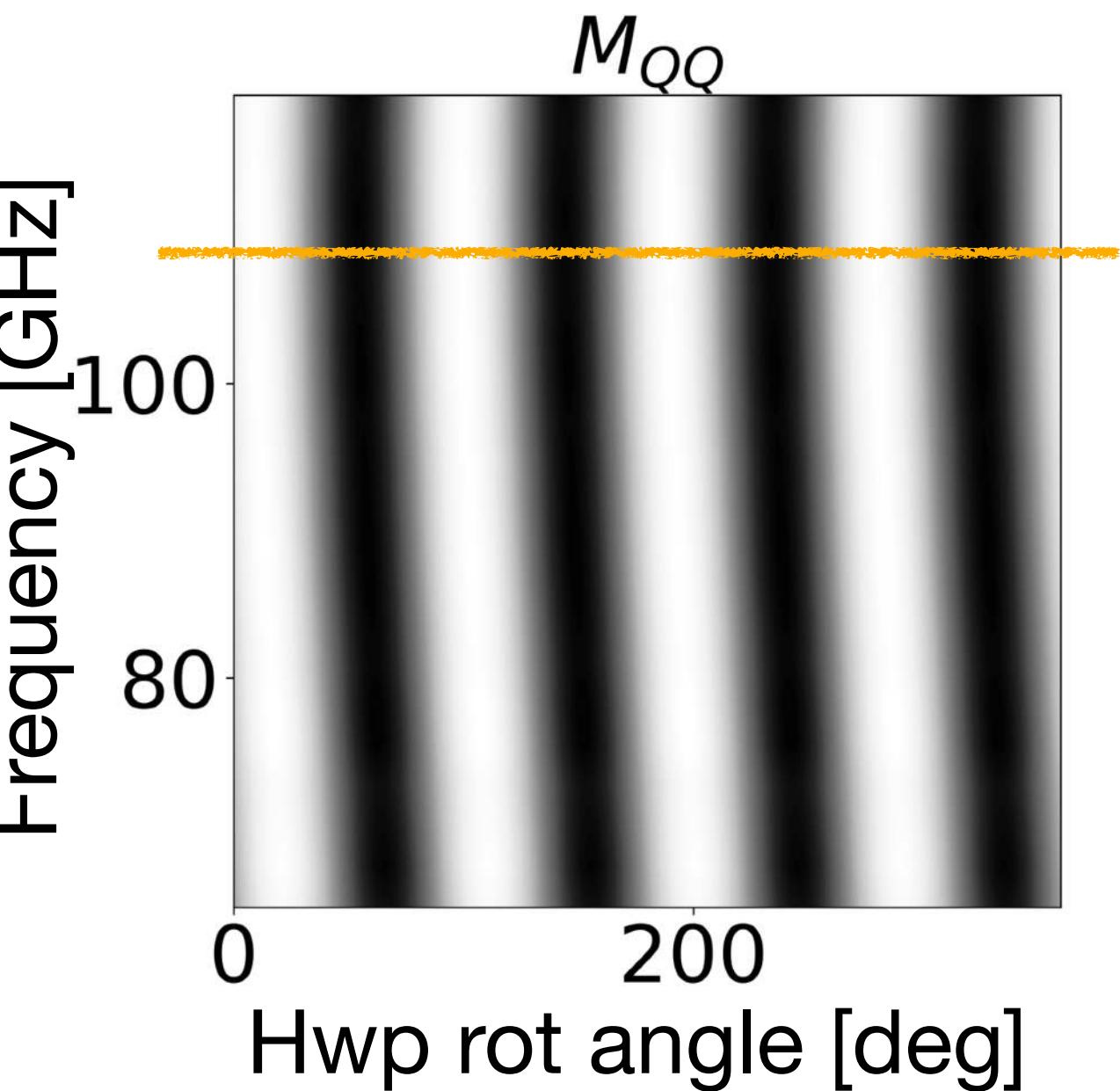
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Reproject on the sky with an **effective ideal hwp**:
calibrate the non-ideal hwp on the ideal one at tod level

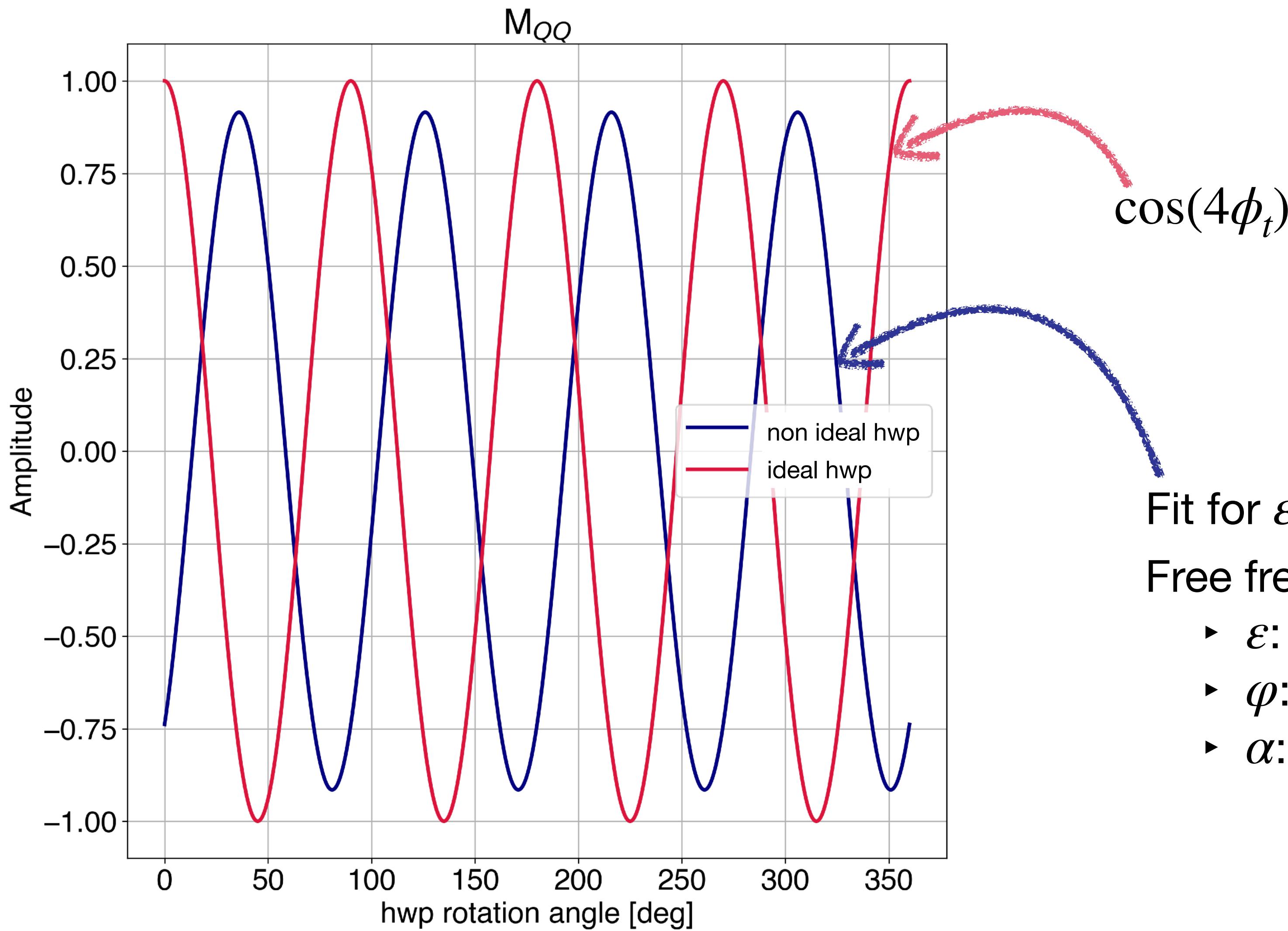
No band pass case

Effective hwp model: fit



No band pass case

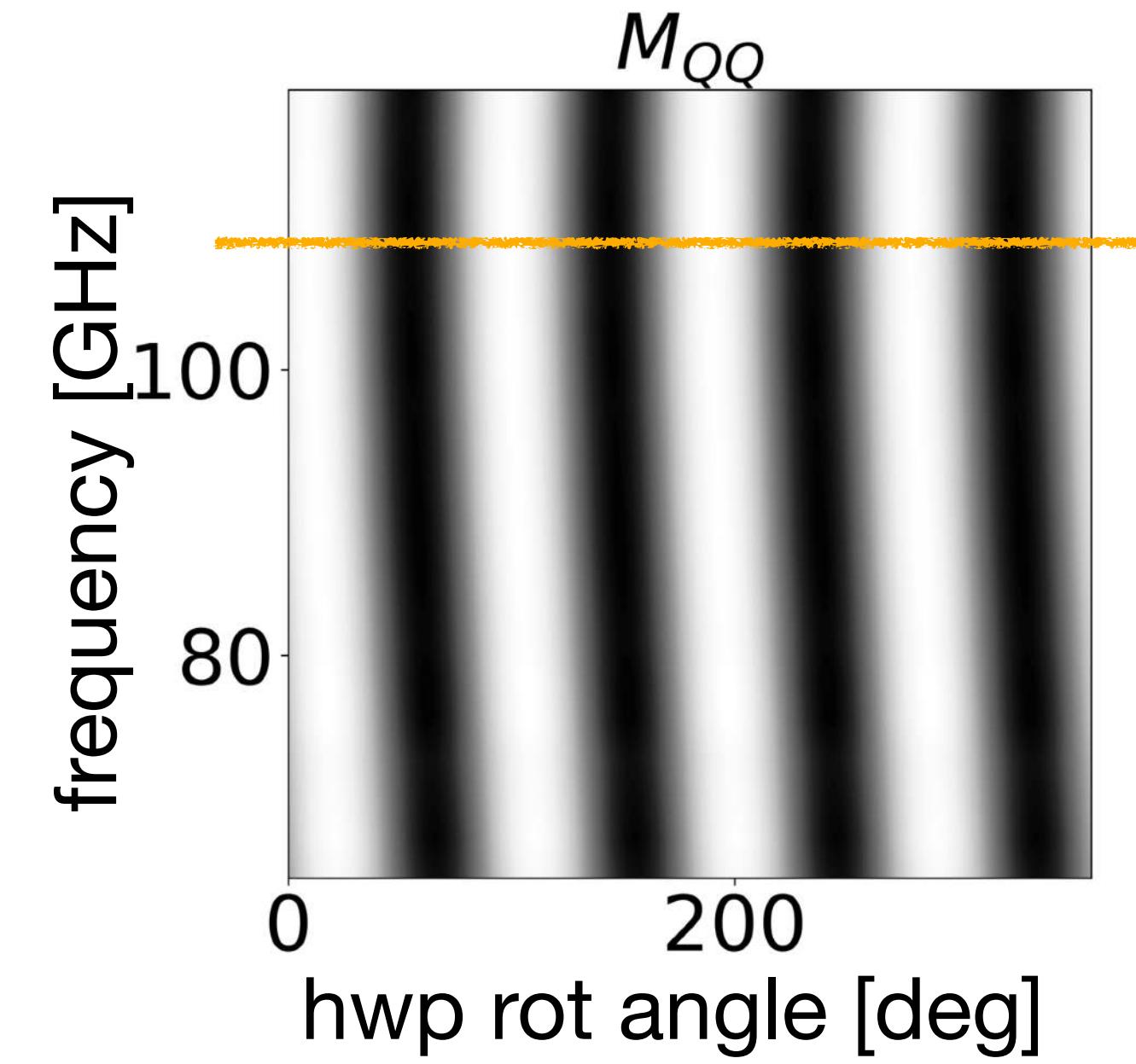
Effective hwp model: fit



Fit for $\varepsilon(\nu) \cos(4\phi_t + \varphi(\nu)) + \alpha(\nu)$

Free frequency-dependent parameters of the fit

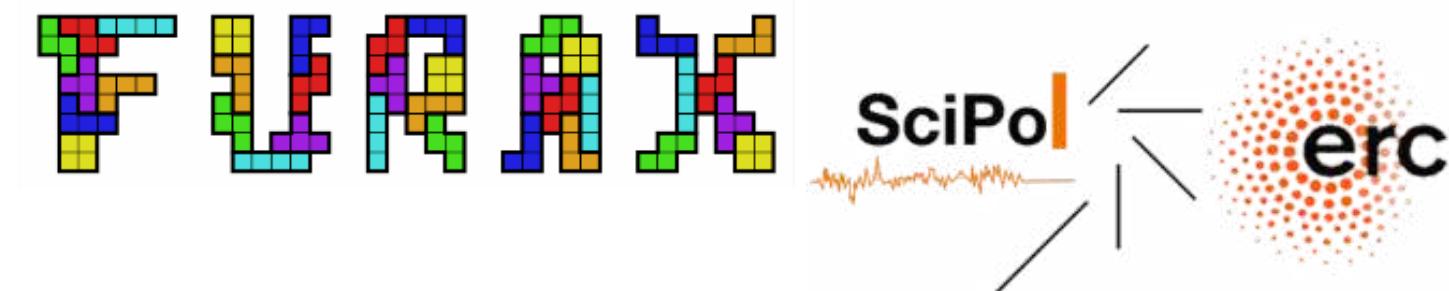
- ε : amplitude
- φ : phase-shift
- α : offset



No band pass case

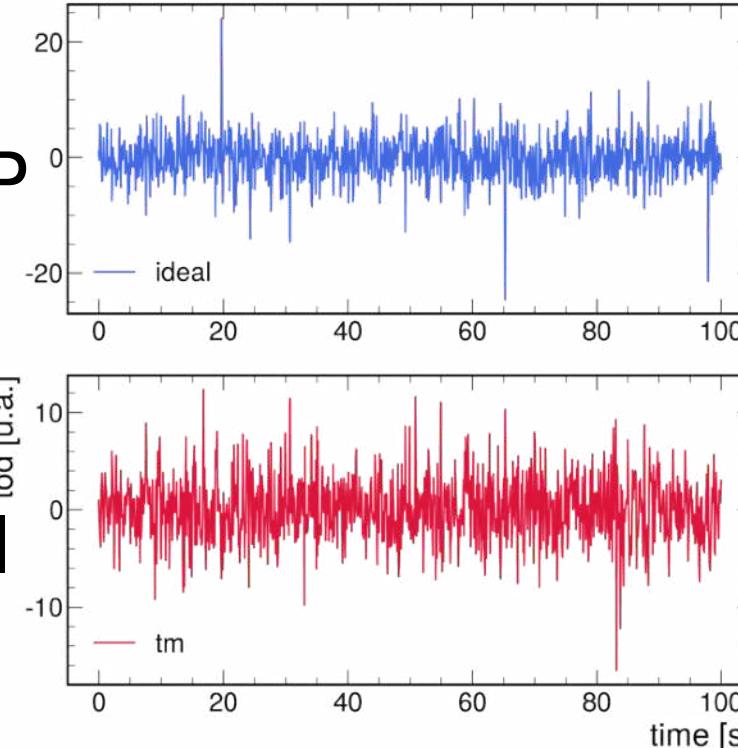
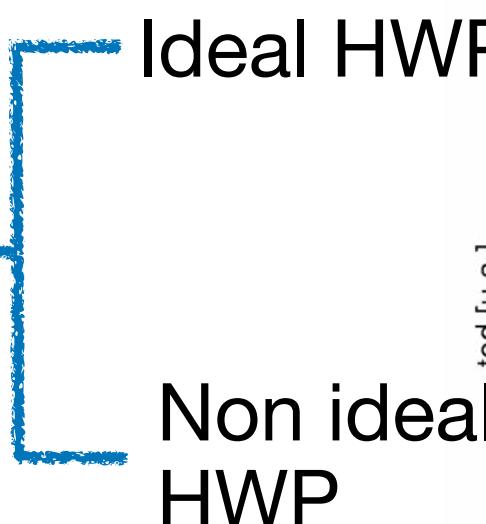
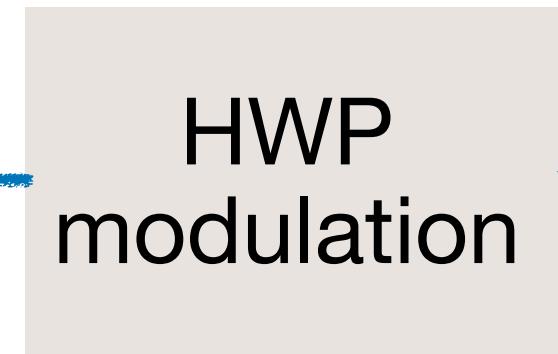
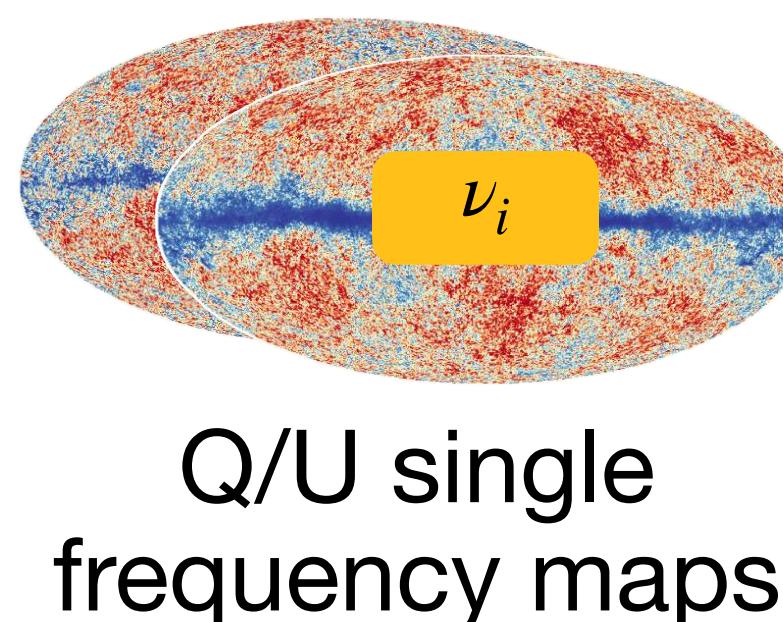
Effective hwp model

Pipeline:



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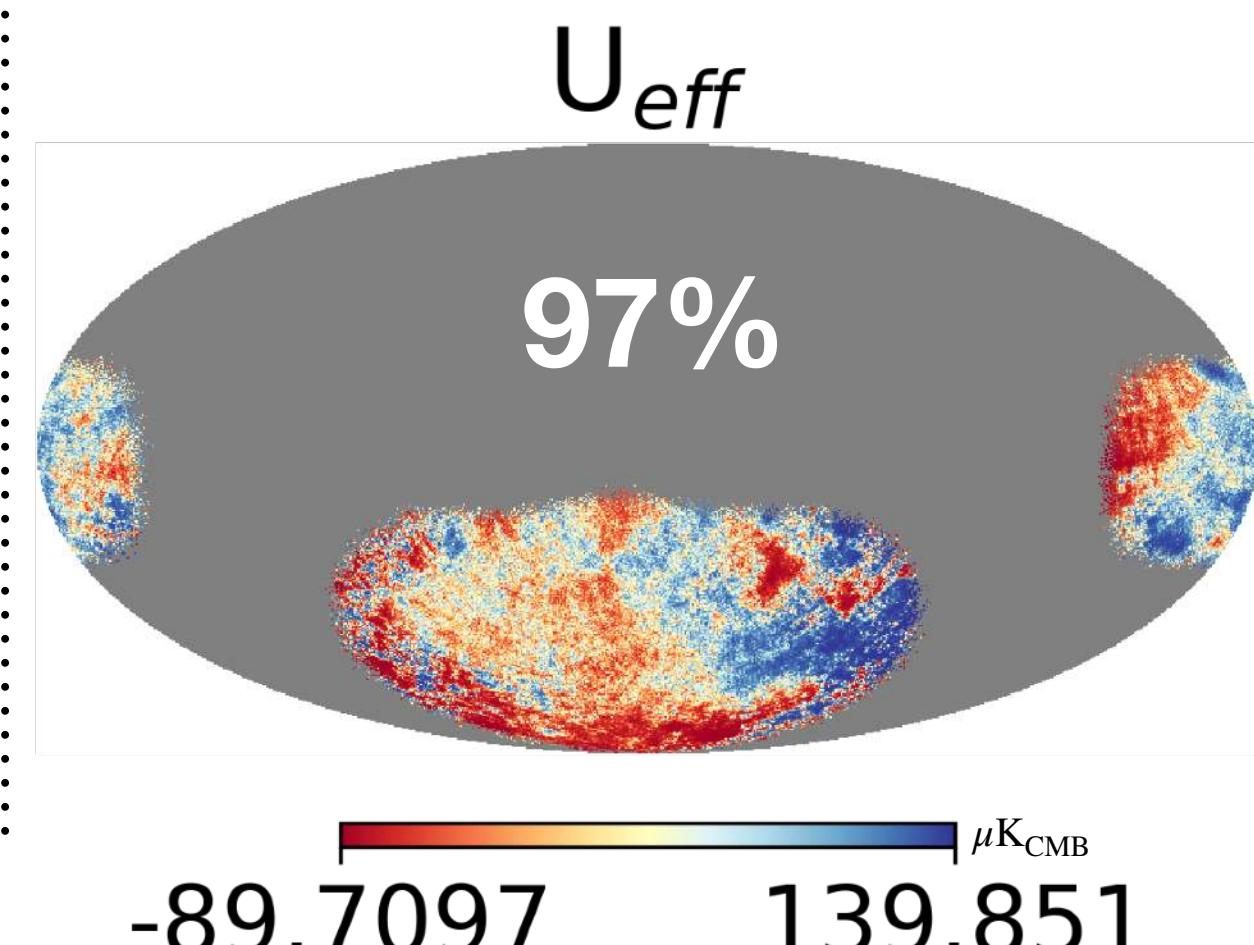
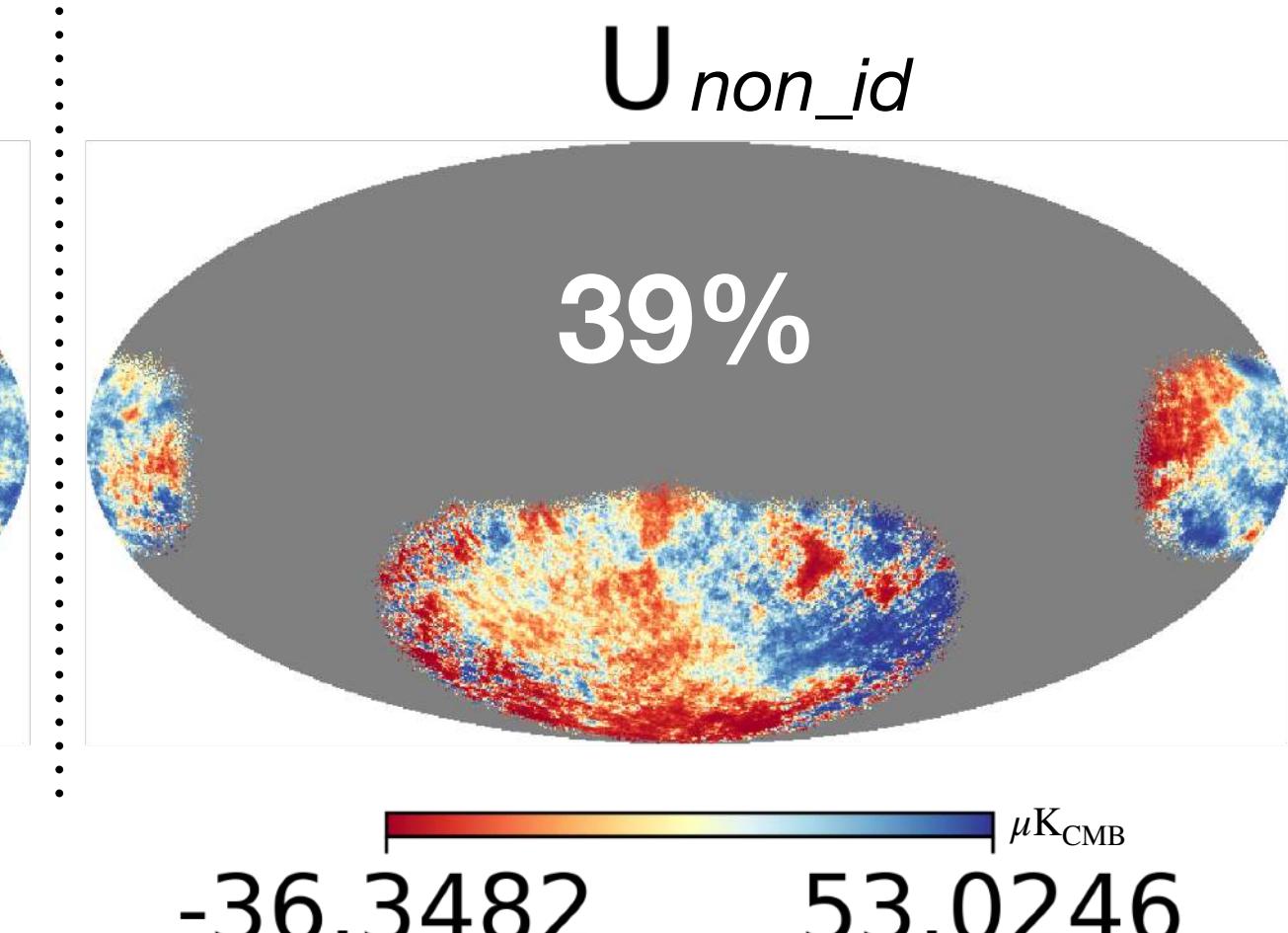
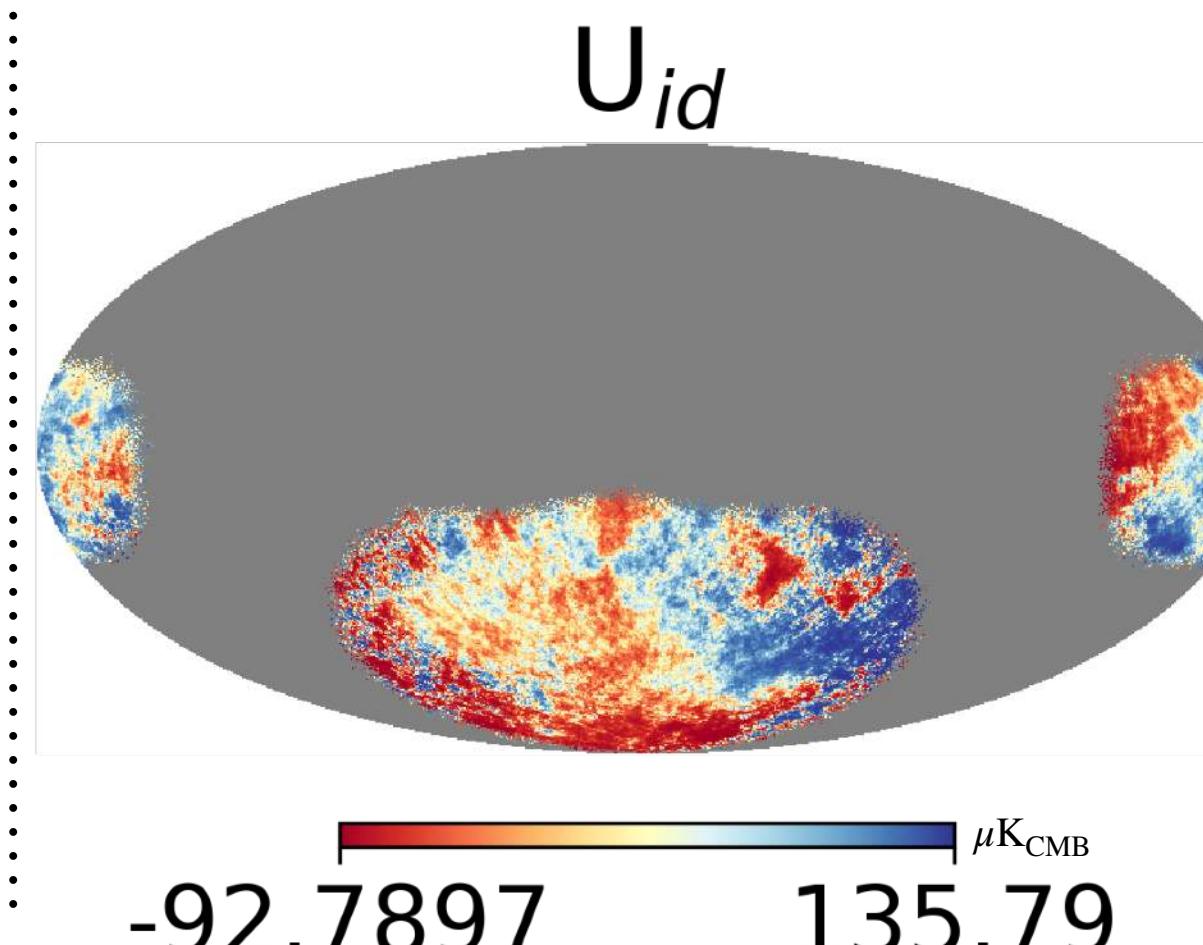
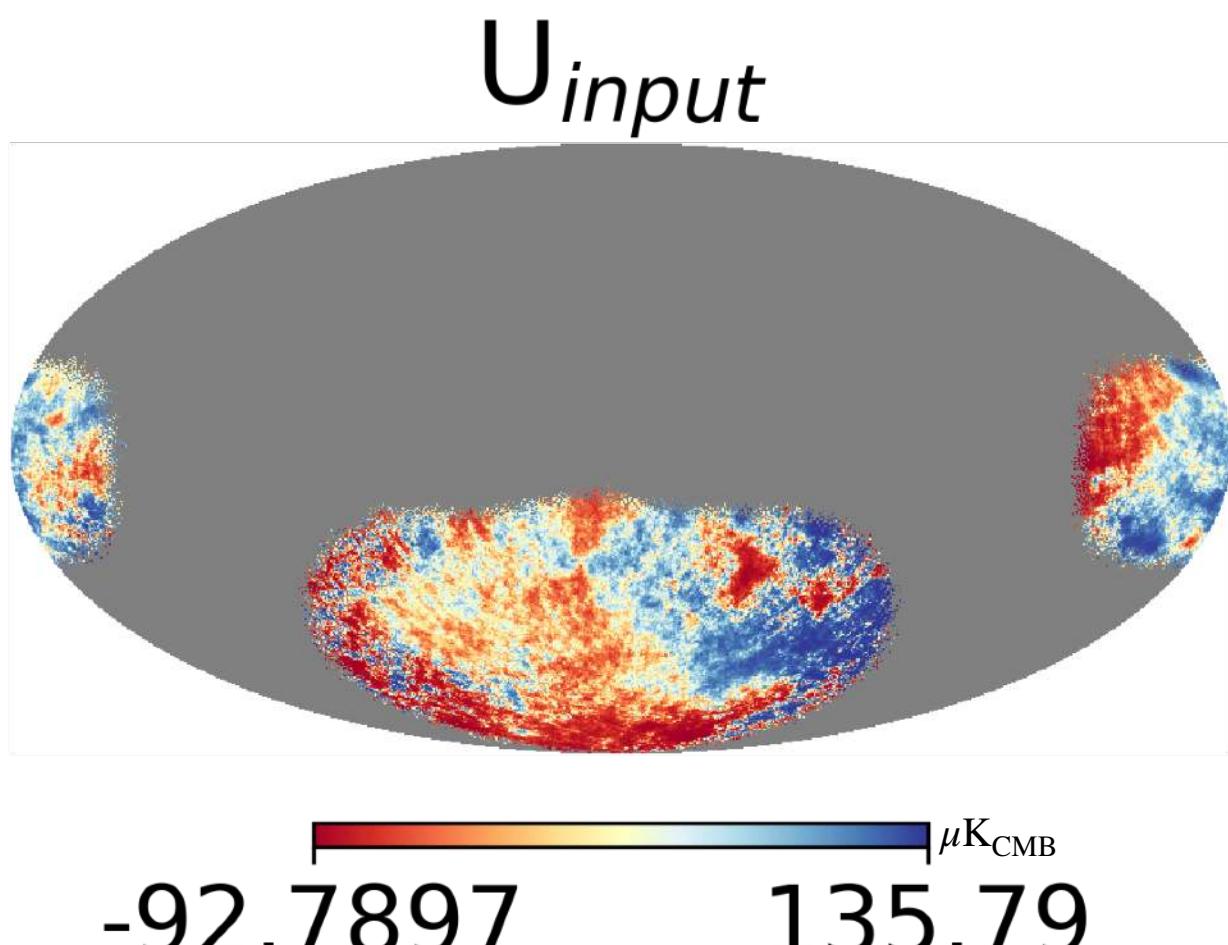
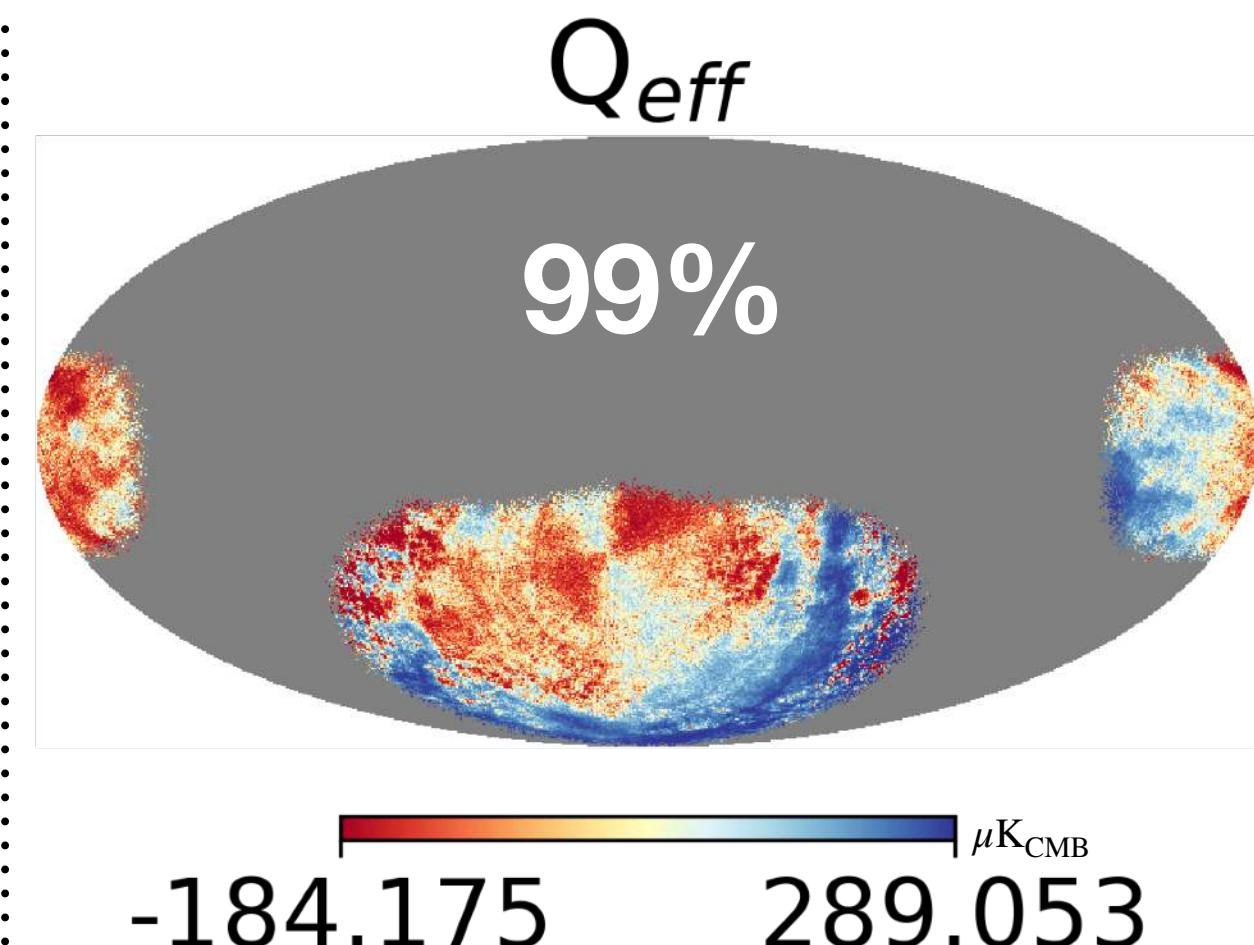
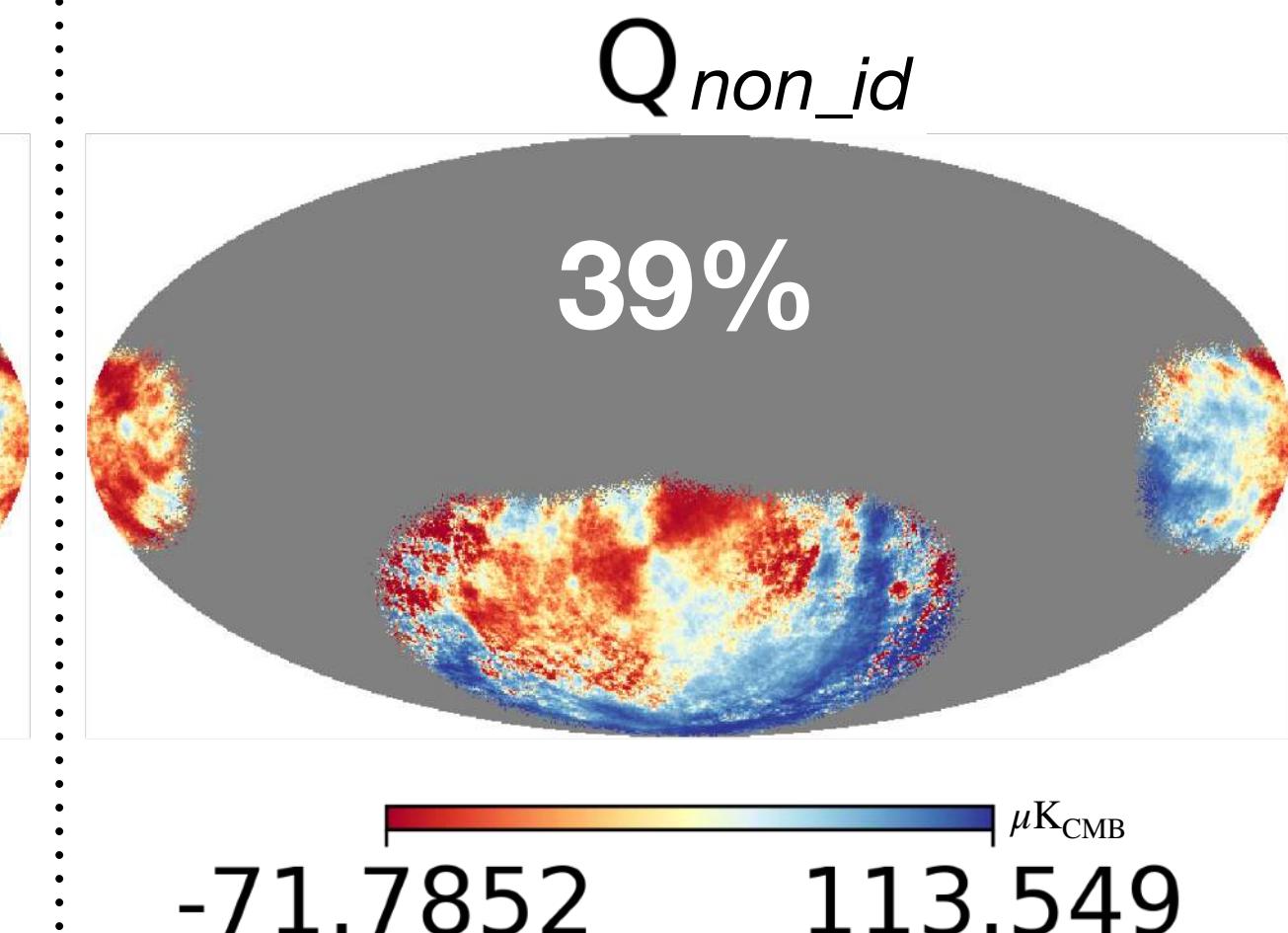
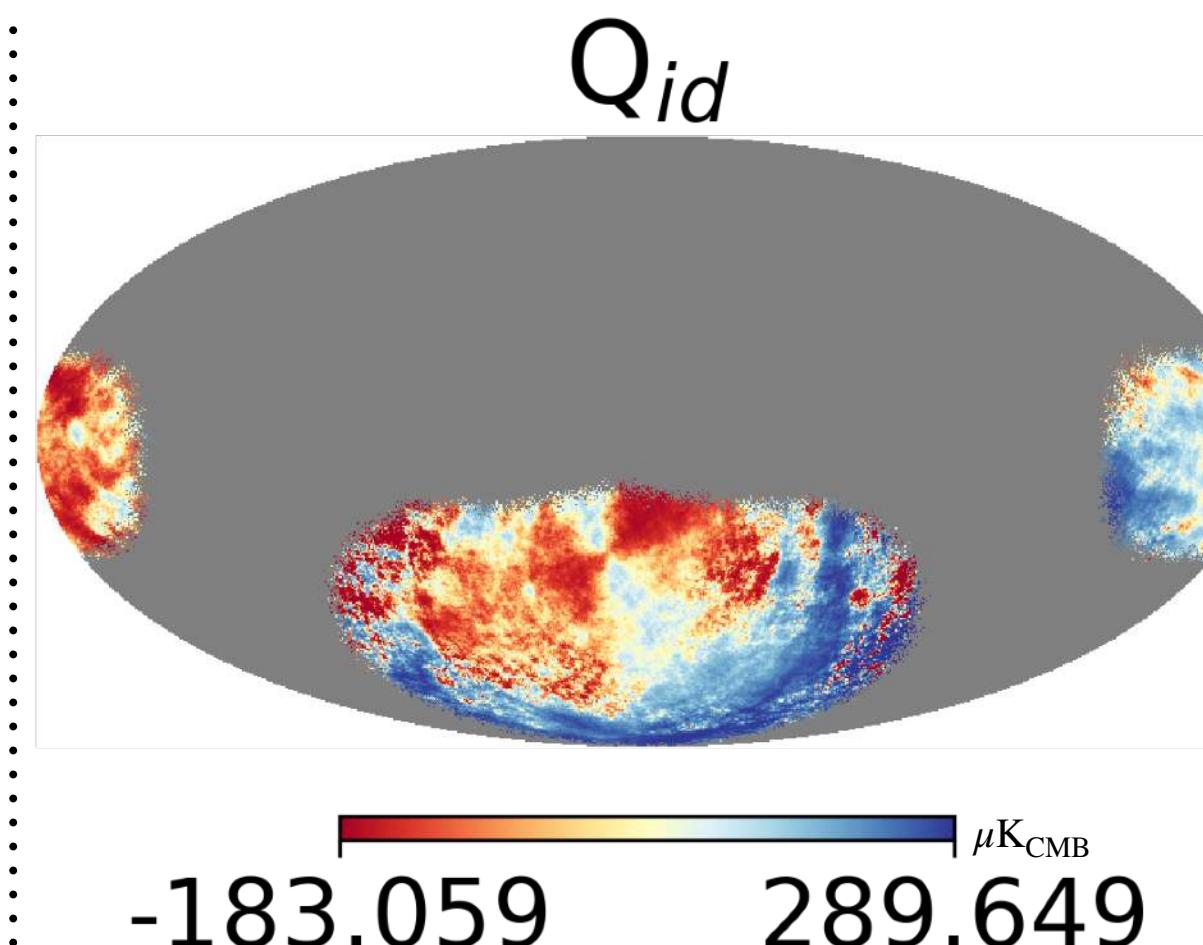
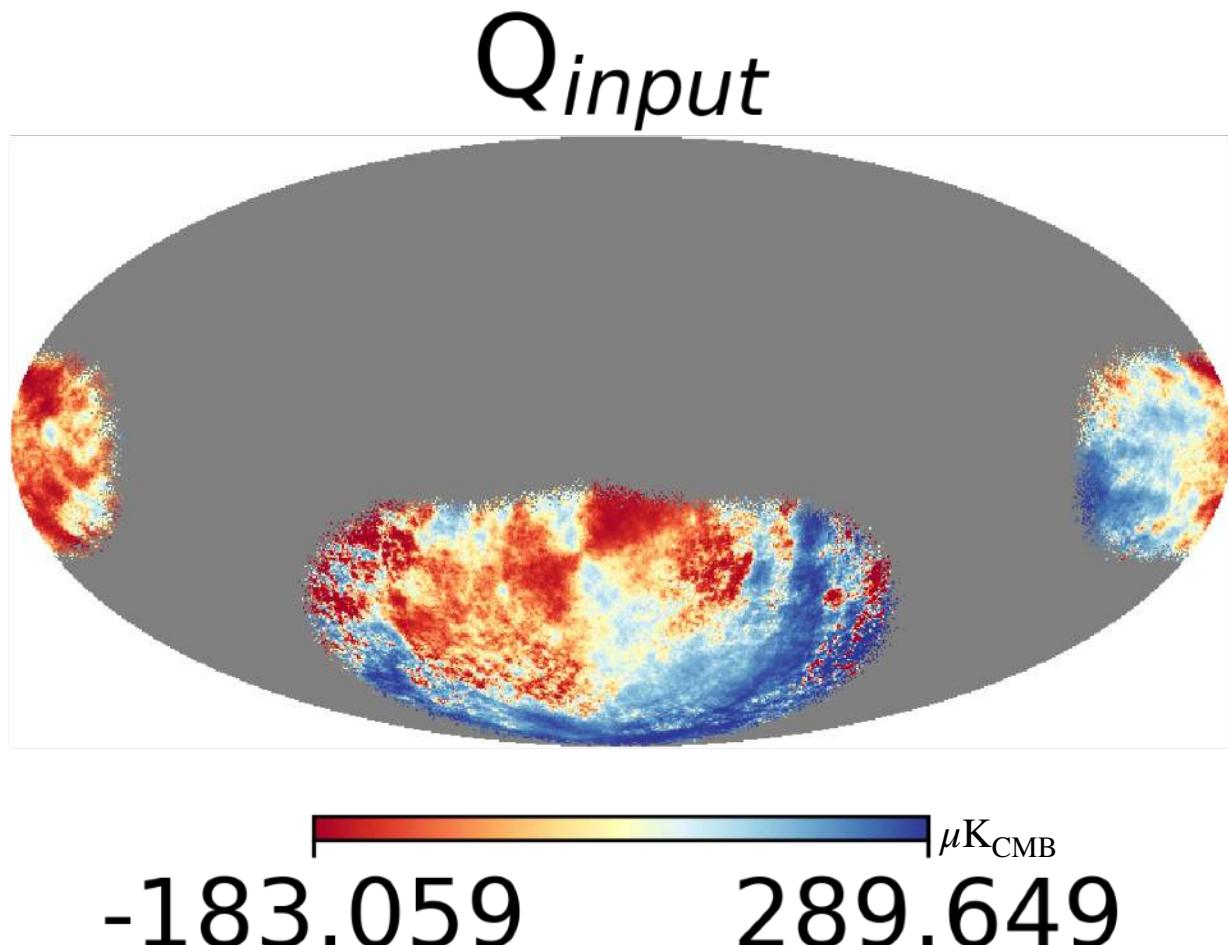
Effective ideal HWP sky reprojection

Q/U residuals
single
frequency
maps

Work in progress

No band pass case

Effective hwp model



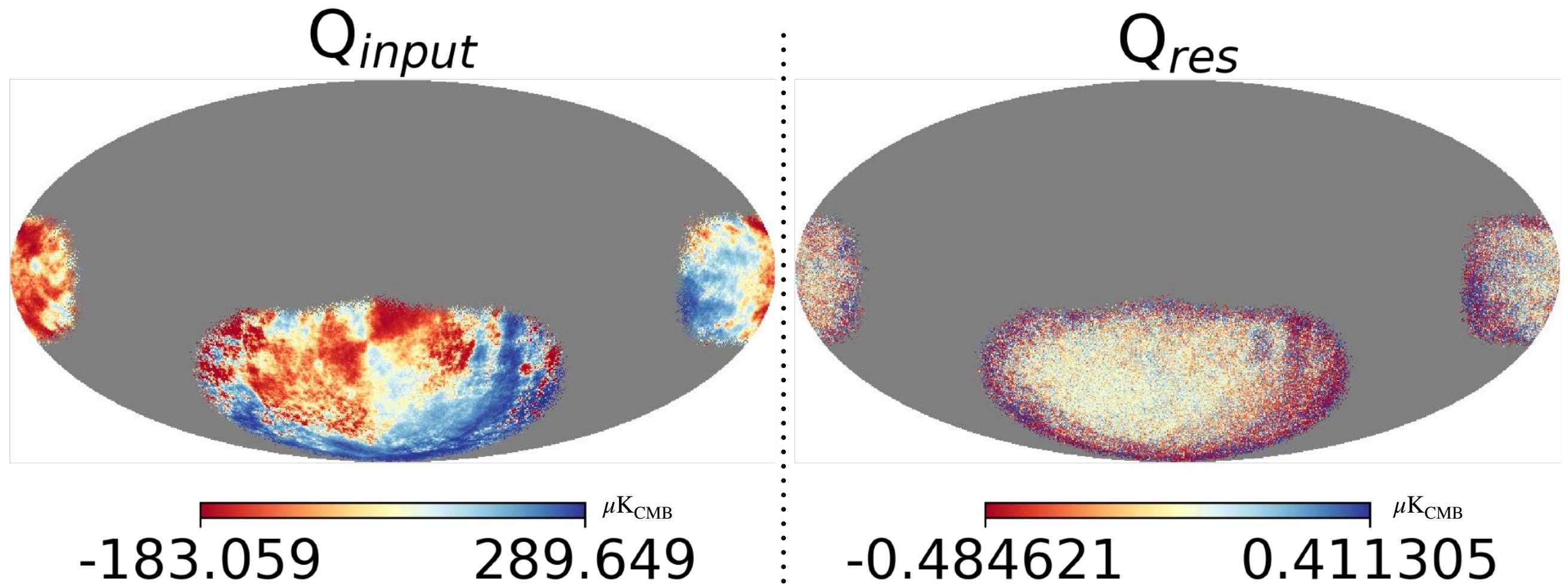
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Effective hwp model

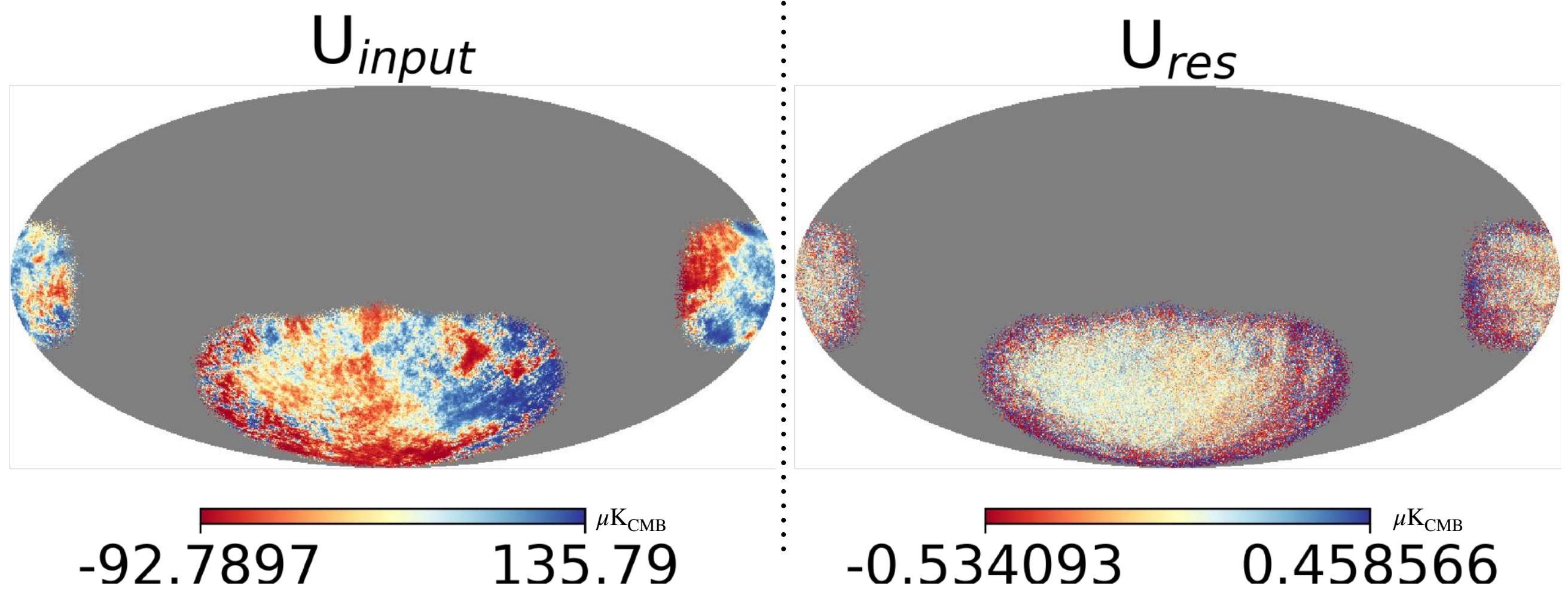
✓ Validation of the **effective** model

Work in progress

UHF2, comp=[cmb+ dust+synch]



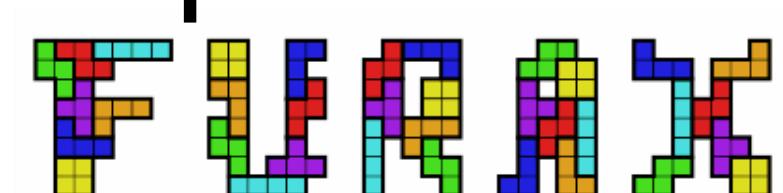
$\text{res} = \text{input} - \text{effective}$



No band pass case

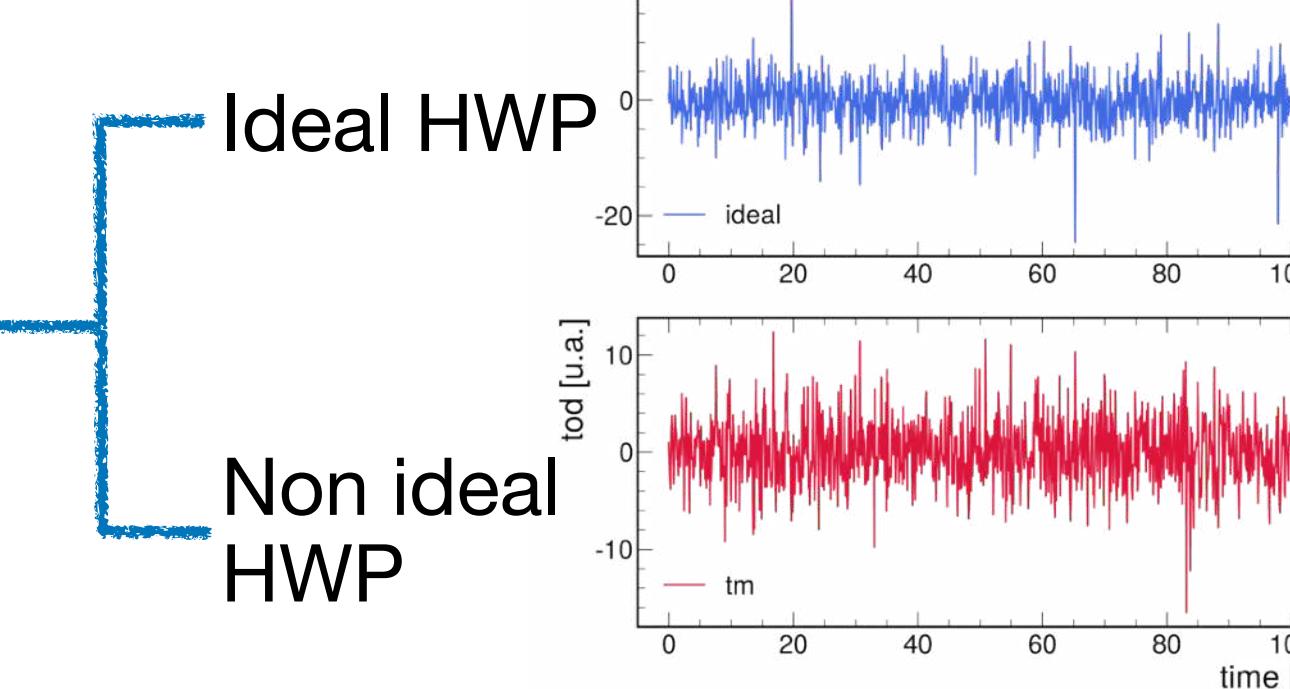
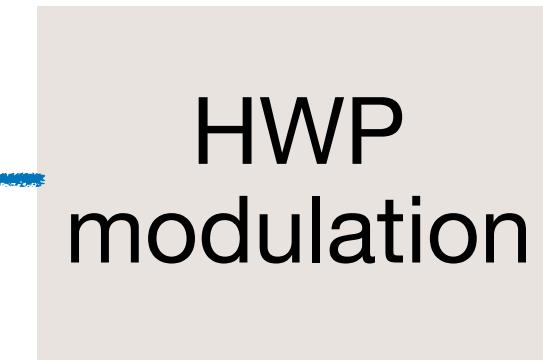
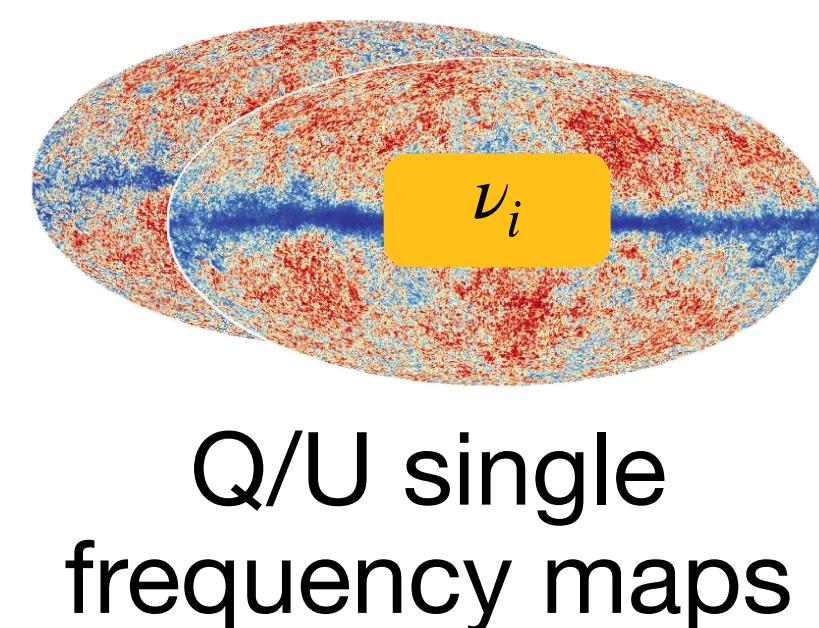
Effective hwp model

Pipeline:



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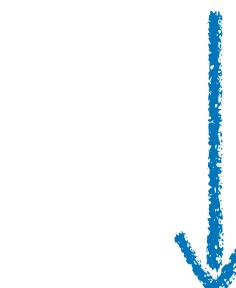


Ideal HWP
Non ideal HWP

Ideal HWP sky reprojection

Effective ideal HWP sky reprojection

Q/U residuals
single
frequency
maps



Component separation

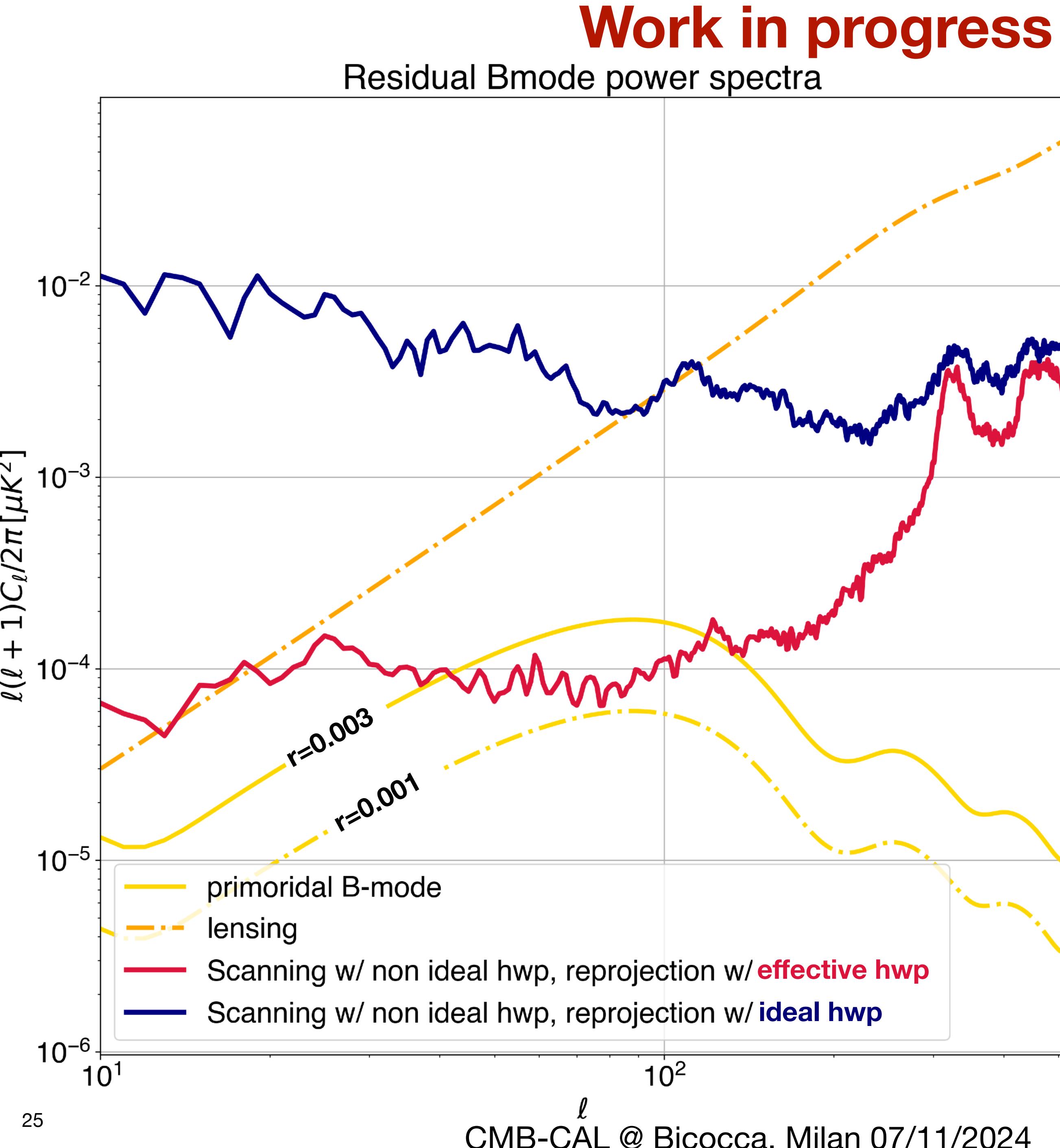
- **fgbuster**
- Frequency bands:
SO-SAT LF1: 27, LF2: 39, MF1: 92, MF2: 148, UHF1: 225.5, UHF2: 286.5
- cmb, dust @ 148 GHz, synchrotron @ 148 GHz

No band pass case

Effective hwp model

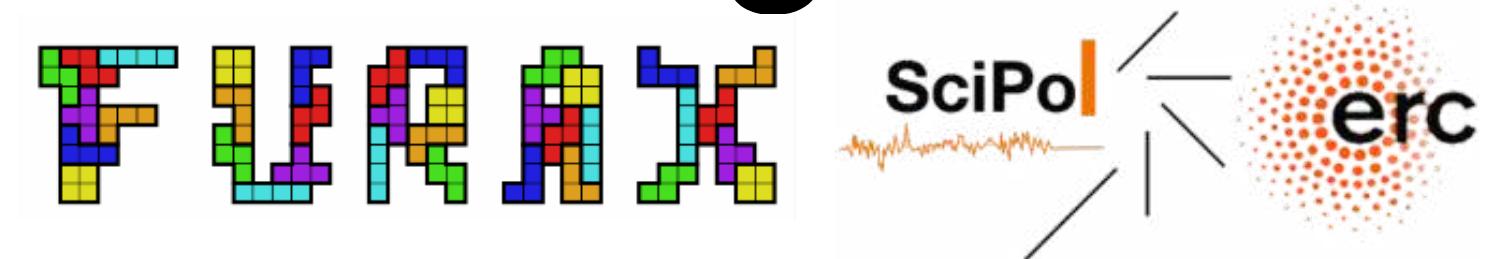
- Frequency bands:
SO-SAT LF1: 27, LF2: 39, MF1: 92, MF2:
148, UHF1: 225.5, UHF2: 286.5
- cmb, dust @ 148 GHz, synchrotron @ 148 GHz

Foreground parameter	β_d	β_{pl}
Input d0s0	1.54	-3.0
Ideal case	1.54	-3.0
Effective case	1.52515383	-3.00331785
Non ideal case (with priors)	1.	-2.86768798



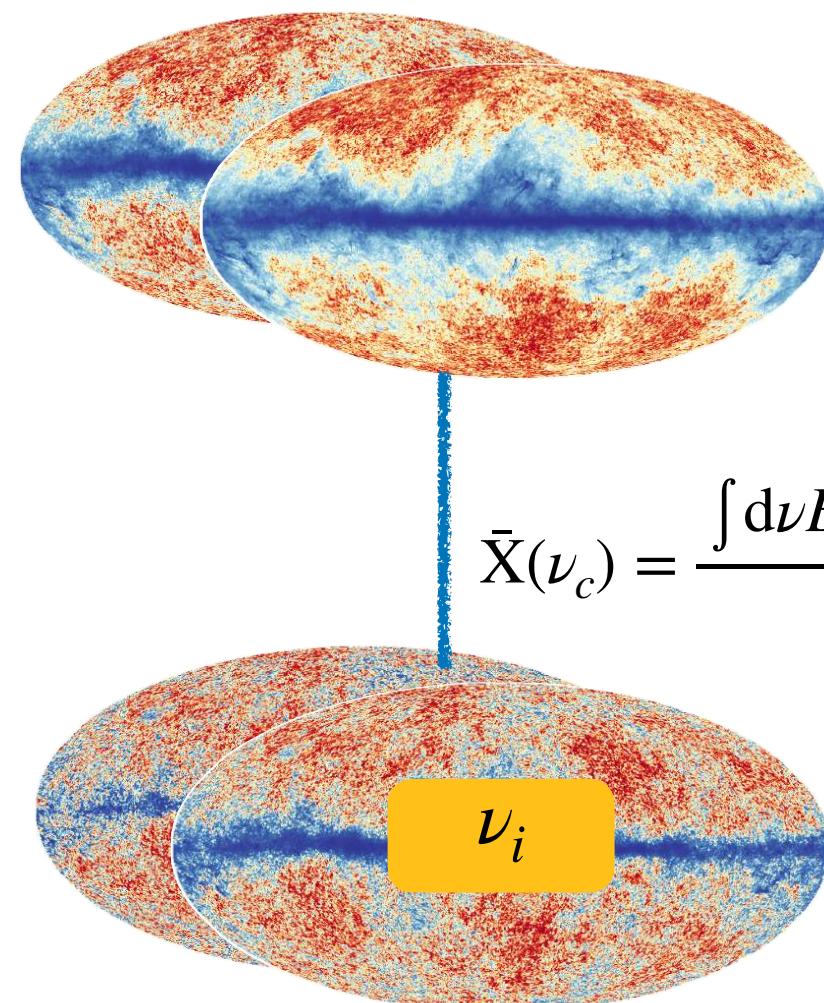
Band-pass integrated frequency case

Pipeline:



- Modularity, extensibility, simplicity:
 $h = \text{pol} @ \text{bandpass} @ \text{hwp} @ \text{mixing_matrix} @ \text{sampling}$
 $\text{sol} = ((h.T @ h).I @ h.T)(\text{tod})$

Q/U integrated frequency maps



HWP modulation

$$\bar{\mathbf{X}}(\nu_c) = \frac{\int d\nu B(\nu, n u_c) \mathbf{X}(\nu)}{\int d\nu B(\nu)}$$

Q/U single frequency maps

- PySm templates:
cmb, dust @ 353 GHz,
synchrotron @ 30 GHz
- No noise
- Scanning strategy: random hits on SO SAT mask

Ideal HWP

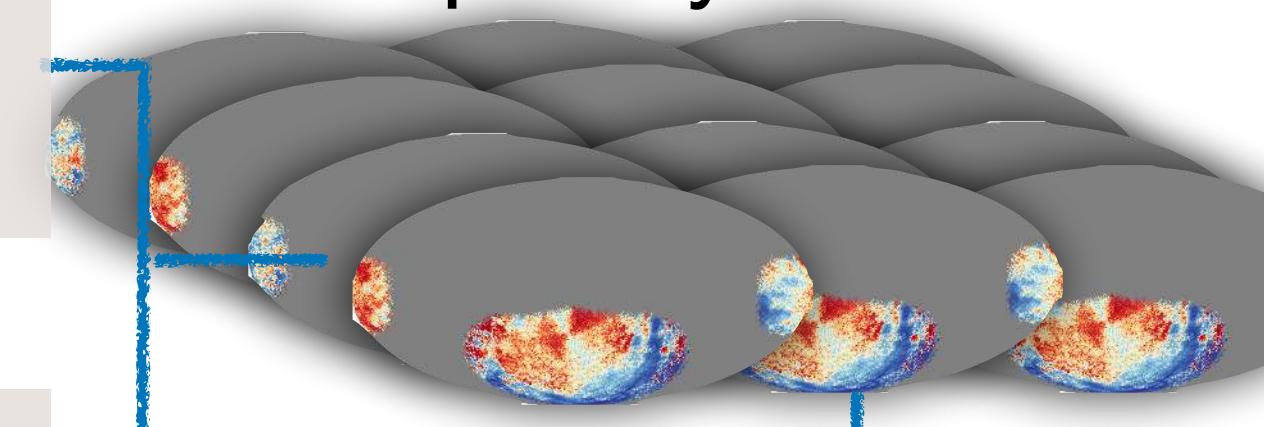
Non ideal HWP

$$\bar{d} = \int d\nu \mathbf{M}_{\text{instr}}(\nu) \mathbf{A}_{\text{SED}}(\nu) s_{\text{sky}} + n$$

Ideal HWP sky reprojection

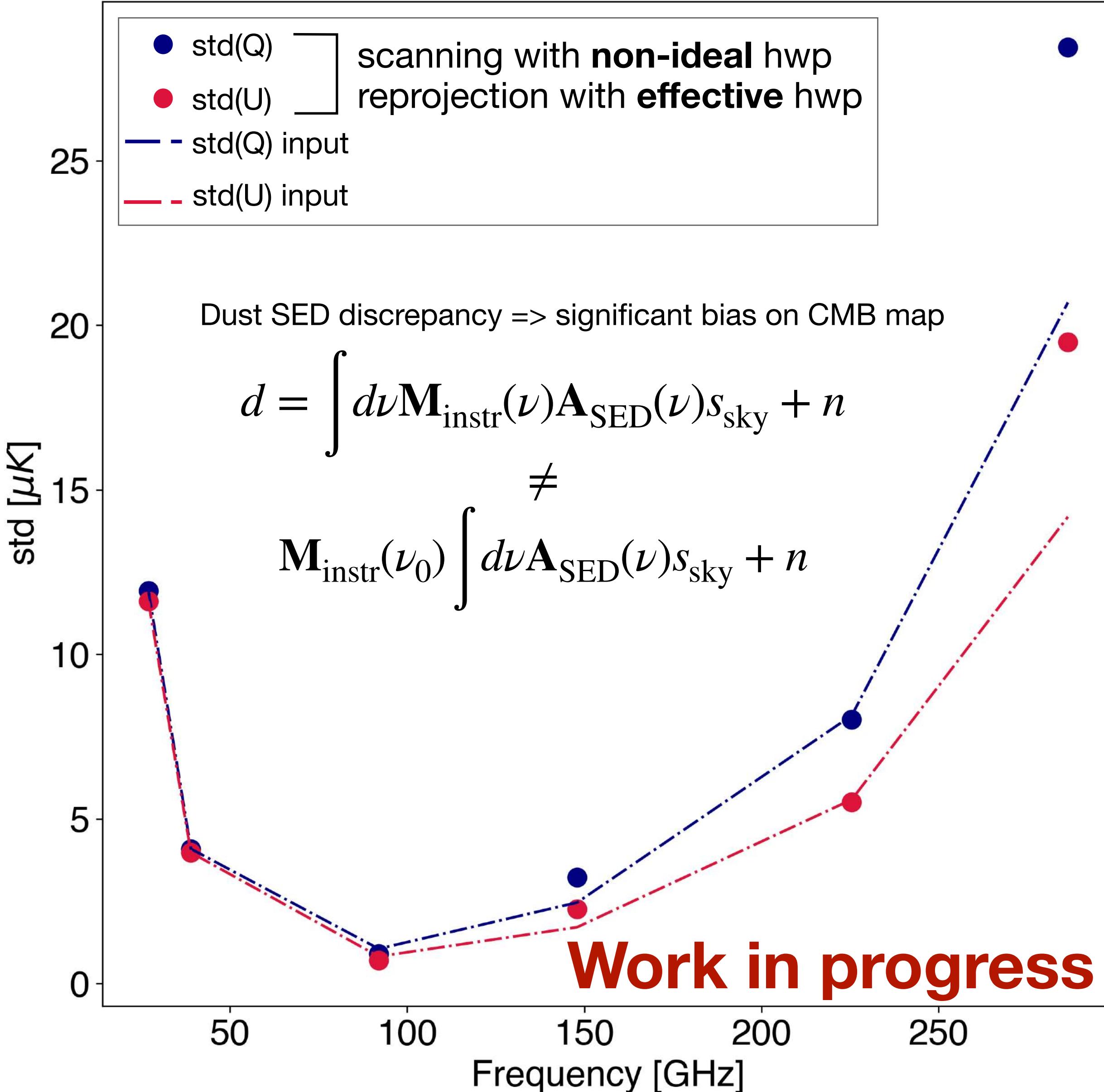
Effective ideal HWP sky reprojection

Q/U frequency maps for 6 frequency channels



Component separation

Band-pass integrated frequency case



- The interplay between instrumental systematics and foreground mitigation calls for a **time-domain component separation**
 - need for **priors**, precise calibration of instruments parameters (*drone*)
- Building of a framework that **includes instrumental systematic effects in the component separation** process : HWP, bandpass
- Extensions:
 - add **more effects** (beams, readout, polarisation angle) (Jost et al. 2023)
 - testing the parametric model against real SO data sets, but also test with LiteBIRD and CMB-S4 configurations if the HWP solution is confirmed
 - paper is in prep.

FURAX

