Polarization angles mitigation in Component Separation for r and β_b Measurements

COSMO-CAL 08/11/2024

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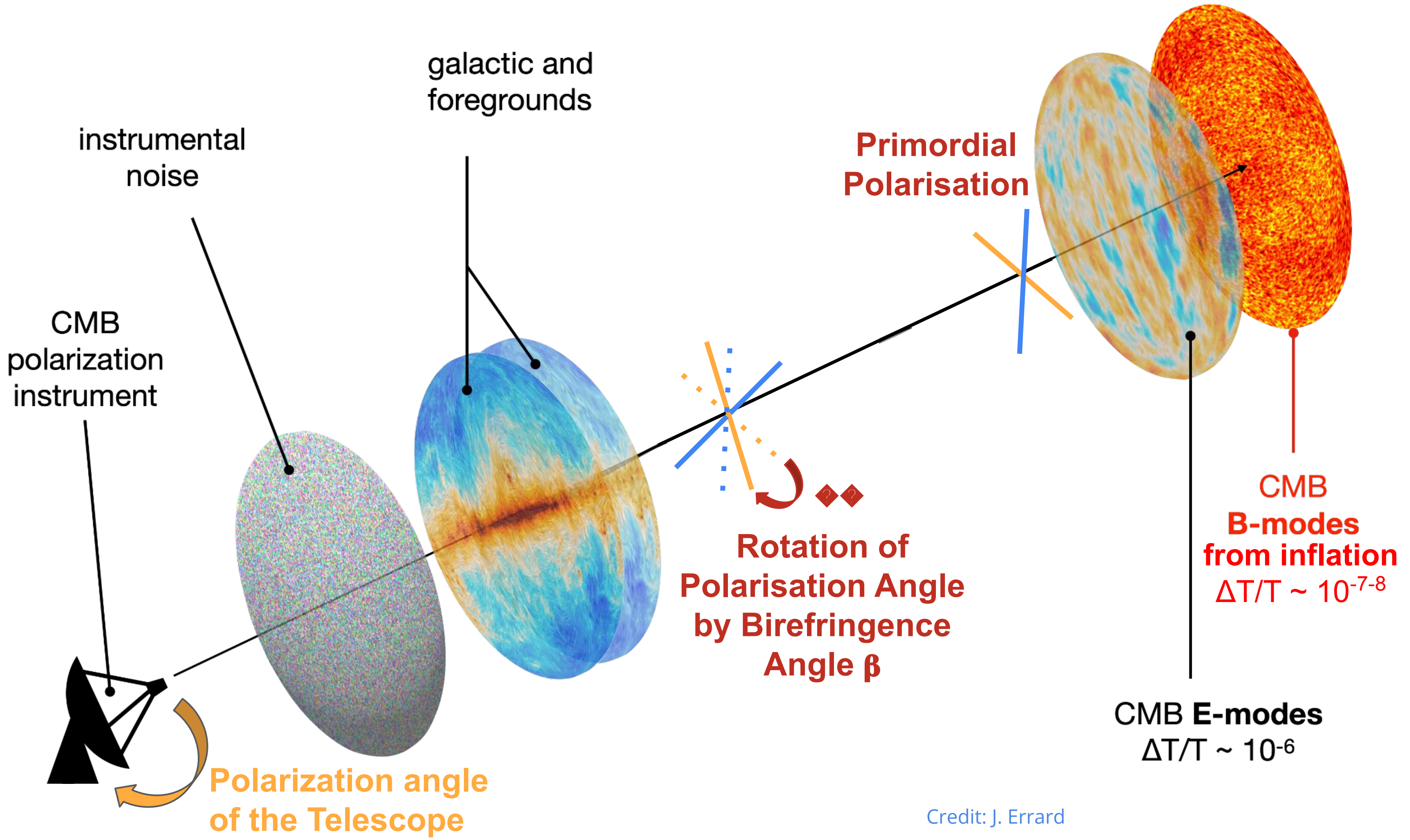
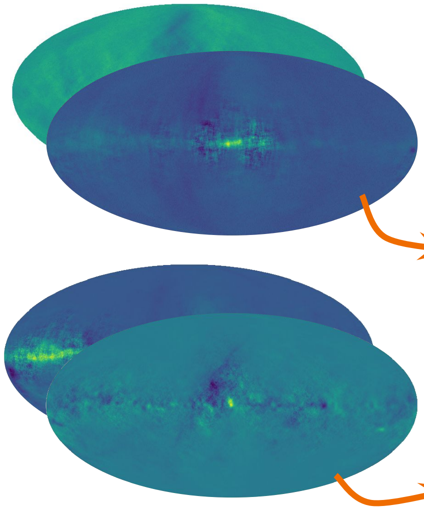


Credit: J. Errard

Parametric Component Separation

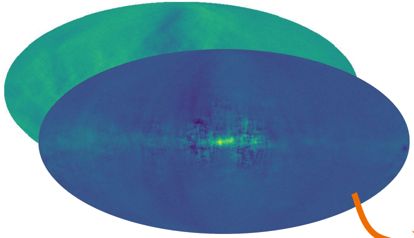


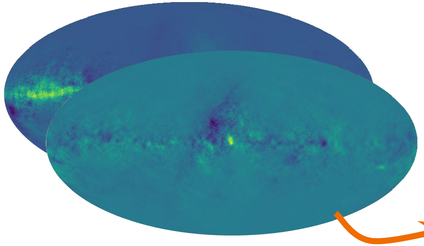
$$\mathbf{d}_p = \begin{pmatrix} Q_1 \\ U_1 \\ \vdots \\ Q_n \\ U_n \end{pmatrix}$$

Input frequency maps

Parametric Component Separation

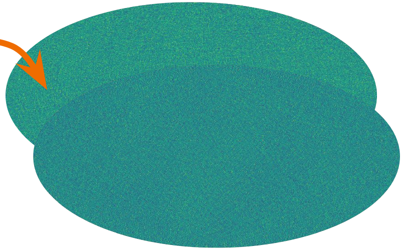
$$\mathbf{d}_p = \quad ? \quad \mathbf{s}_p$$





$$\begin{pmatrix} Q_1 \\ U_1 \\ \vdots \\ Q_n \\ U_n \end{pmatrix}$$

$$\begin{pmatrix} Q^{CMB} \\ U^{CMB} \\ Q^d \\ U^d \\ Q^s \\ U^s \end{pmatrix}$$



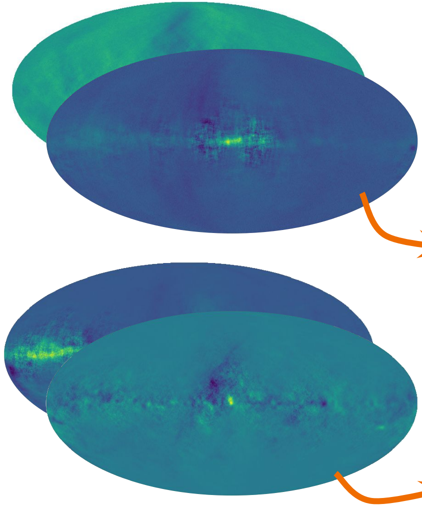
Input frequency maps

Component maps

Parametric Component Separation

$$\mathbf{d}_p = \quad ? \quad \mathbf{s}_p + \mathbf{n}_p$$

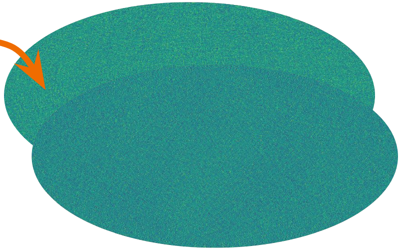
noise



$$\begin{pmatrix} Q_1 \\ U_1 \\ \vdots \\ Q_n \\ U_n \end{pmatrix}$$

Input frequency maps

$$\begin{pmatrix} Q^{CMB} \\ U^{CMB} \\ Q^d \\ U^d \\ Q^s \\ U^s \end{pmatrix}$$



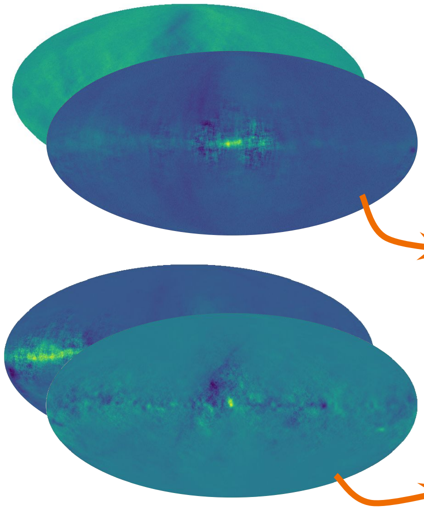
Component maps

Parametric Component Separation

$$\mathbf{d}_p = \mathbf{A}(\beta_{\mathrm{fg}})\mathbf{s}_p + \mathbf{n}_p$$

noise

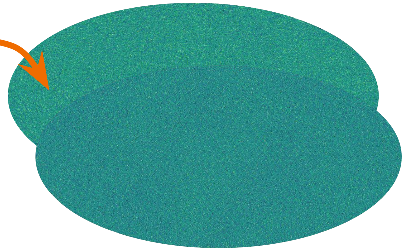
The Mixing Matrix



$$\begin{pmatrix} Q_1 \\ U_1 \\ \vdots \\ Q_n \\ U_n \end{pmatrix}$$

Input frequency maps

$$\begin{pmatrix} Q^{CMB} \\ U^{CMB} \\ Q^d \\ U^d \\ Q^s \\ U^s \end{pmatrix}$$

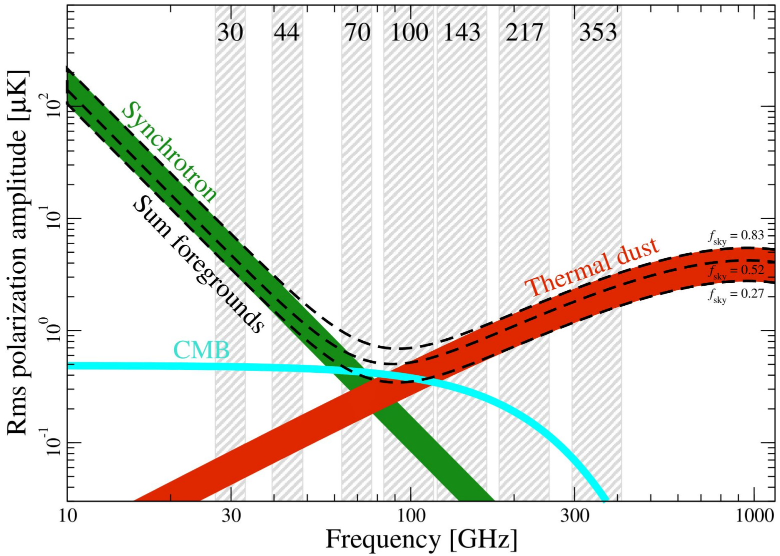


Component maps

The Mixing Matrix

$$\mathbf{d}_p = \mathbf{A}(\beta_{\mathrm{fg}})\mathbf{s}_p + \mathbf{n}_p$$

$$\boldsymbol{A}(\{\beta_{fg}\}) = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{CMB}} \underbrace{\begin{matrix} A_1^d & 0 \\ 0 & A_1^d \\ \vdots & \vdots \\ A_n^d & 0 \\ 0 & A_n^d \end{matrix}}_{\text{Dust } T_d,\, \beta_d} \underbrace{\begin{matrix} A_1^s & 0 \\ 0 & A_1^s \\ \vdots & \vdots \\ A_n^s & 0 \\ 0 & A_n^s \end{matrix}}_{\text{Synchrotron } \beta_s}$$



Source: Planck Collaboration 2018

Assumed foreground emission laws:

- **modified black-body** for dust
- **power law** for synchrotron

Variation across the sky is possible (Errard et al. 2019)

Spectral Likelihood

Using the “spectral likelihood” (Stompor et al 2008) we only have to estimate foreground parameters:

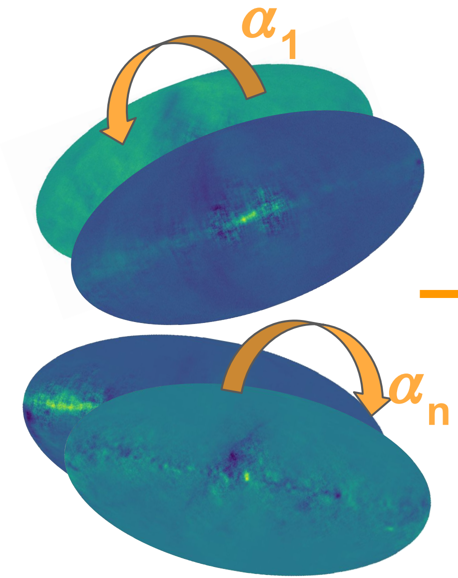
$$-2\ln\mathcal{L}_{\rm spec}(\beta_{\rm fg}) = {\rm cst} - (\mathbf{A}^{\rm t}\mathbf{N}^{-1}\mathbf{d})^{\rm t}(\mathbf{A}^{\rm t}\mathbf{N}^{-1}\mathbf{A})^{-1}(\mathbf{A}^{\rm t}\mathbf{N}^{-1}\mathbf{d})$$

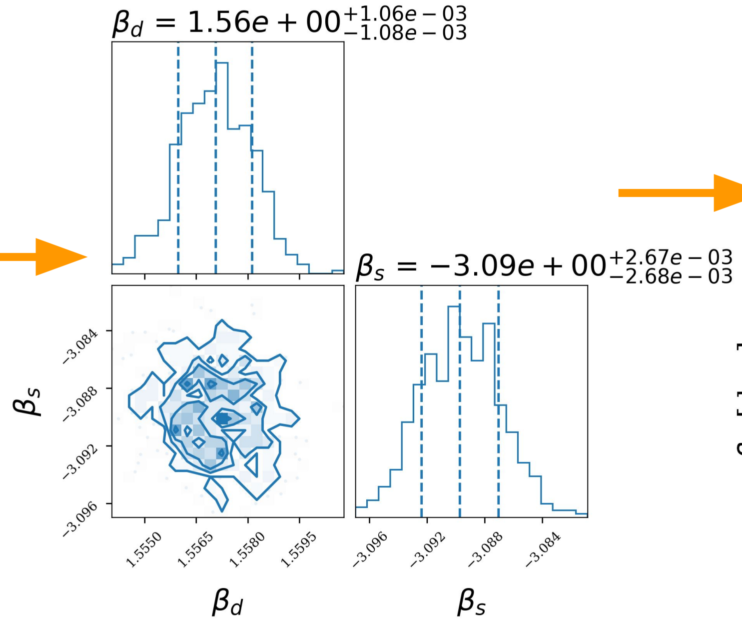
The component maps are then given by:

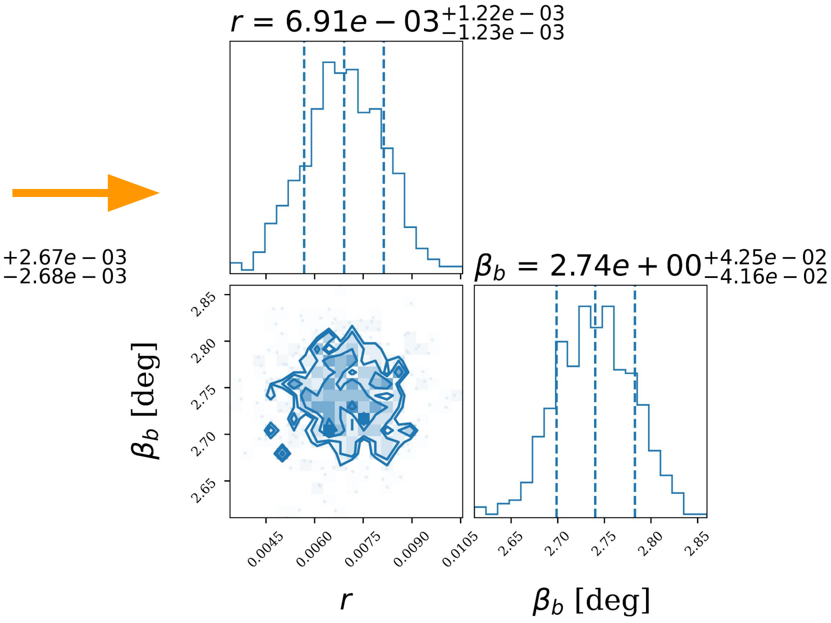
$$\hat{\mathbf{s}} = (\hat{\mathbf{A}}^t\mathbf{N}^{-1}\hat{\mathbf{A}})^{-1}\hat{\mathbf{A}}^t\mathbf{N}^{-1}\mathbf{d}$$

But what about instrumental parameters?

The Effect of Uncontrolled Polarization Angles (a Toy Model)







Non-zero relative polarisation angles

Adding Polarization Angle as a Parameter

$$\mathbf{d}_p = \mathbf{X}(\{\alpha\})\mathbf{A}(\beta_{\mathrm{fg}})\mathbf{s}_p + \mathbf{n}_p$$

$$\boldsymbol{X}(\{\alpha_1, ..., \alpha_{n_f}\}) = \begin{pmatrix} \cos(2\alpha_1) & \sin(2\alpha_1) & & & 0 \\ -\sin(2\alpha_1) & \cos(2\alpha_1) & & & \\ & & \ddots & & \\ & & & \cos(2\alpha_{n_f}) & \sin(2\alpha_{n_f}) \\ 0 & & & -\sin(2\alpha_{n_f}) & \cos(2\alpha_{n_f}) \end{pmatrix}$$

- Relative angles can be retrieved thanks to frequency dependence in mixing matrix!
- What about the absolute angle?

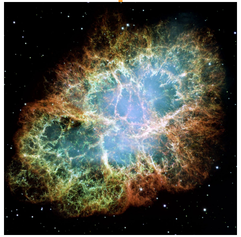
Include Calibration Priors in Component Separation

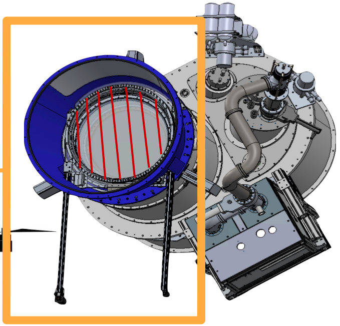
Adding priors from calibration (e.g. wire-grid, drone, Tau-A, cube-sat ...) lifts the degeneracy between birefringence and absolute polarization angle.

Generalized component-separation → CMB map cleaned and corrected for miscalibration.

$$-2\ln\mathcal{L}_{\mathrm{spec}}(\beta_{\mathrm{fg}},\{\alpha\}) = \mathrm{cst} - (\mathbf{\Lambda}^{\mathrm{t}}\mathbf{N}^{-1}\mathbf{d})^{\mathrm{t}}(\mathbf{\Lambda}^{\mathrm{t}}\mathbf{N}^{-1}\mathbf{\Lambda})^{-1}(\mathbf{\Lambda}^{\mathrm{t}}\mathbf{N}^{-1}\mathbf{d})$$

$$+\sum_{\mathrm{i}=1}^{\mathrm{n_f}}\frac{(\alpha_{\mathrm{i}}-\alpha_{\mathrm{i}}^{\mathrm{cal}})^2}{\sigma^2_{\alpha_{\mathrm{i}}^{\mathrm{cal}}}}$$





Wire grid

Cosmological Likelihood

- **Include polarization angle rotation in cosmological likelihood:** corrects for remaining absolute angle (à la self calibration Keating et al. 2012) after component separation
- No E→B leakage: r is retrieved
- Calibration priors ⇒ remaining absolute angle = birefringence angle

$$\mathbf{C}_\ell^{\text{theory}}(r,\beta_b) = \mathbf{R}(\beta_b)\begin{pmatrix} C_\ell^{EE,\,\text{prim}} & 0 \\ 0 & r.C_\ell^{BB,\,\text{prim}} + C_\ell^{BB,\,\text{lens}} \end{pmatrix}\mathbf{R}^{-1}(\beta_b) + \mathbf{C}_\ell^{\text{noise}}$$

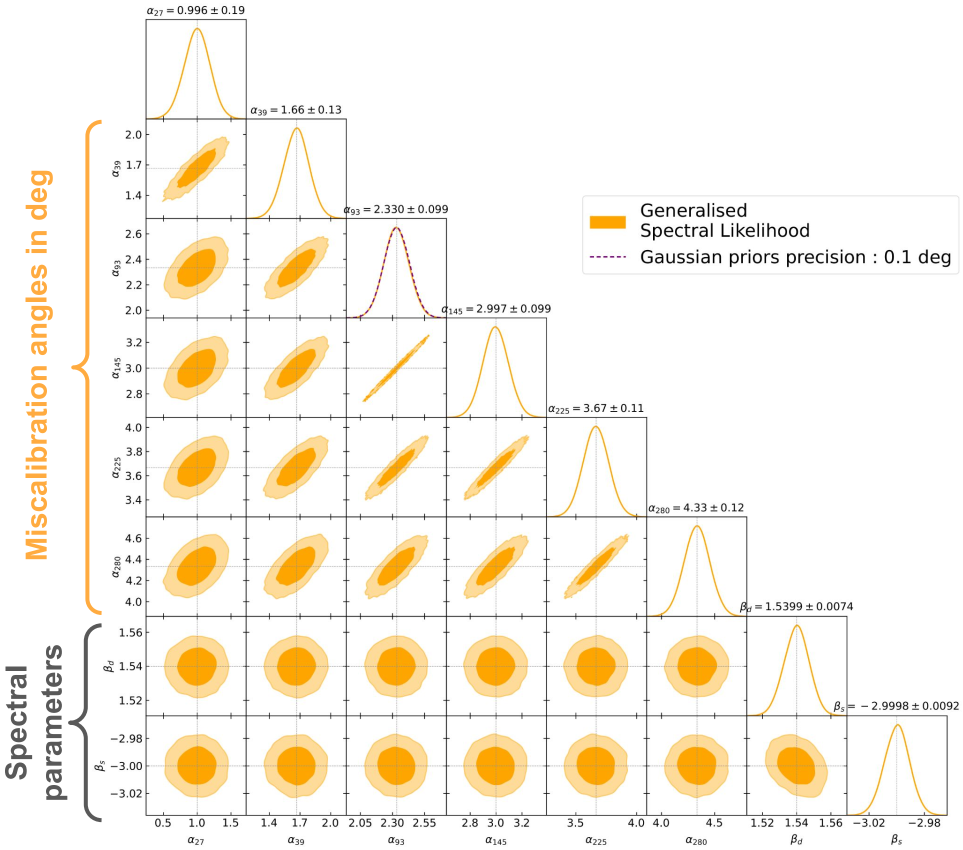
$$-2\log\mathcal{L}^{\text{cosmo}}(r,\beta_b) = \sum_\ell f_{\text{sky}}(2\ell+1)\left[\mathbf{C}_\ell^{\text{theory}\,-1}(r,\beta_b)\mathbf{D}_\ell + \log(|\mathbf{C}_\ell^{\text{theory}}(r,\beta_b)|)\right]$$

One Prior Case

Test case in Jost et al. PRD 2023: SO SAT-like survey (6 frequency channels)

Spectral parameters are correctly estimated (d0s0 input)

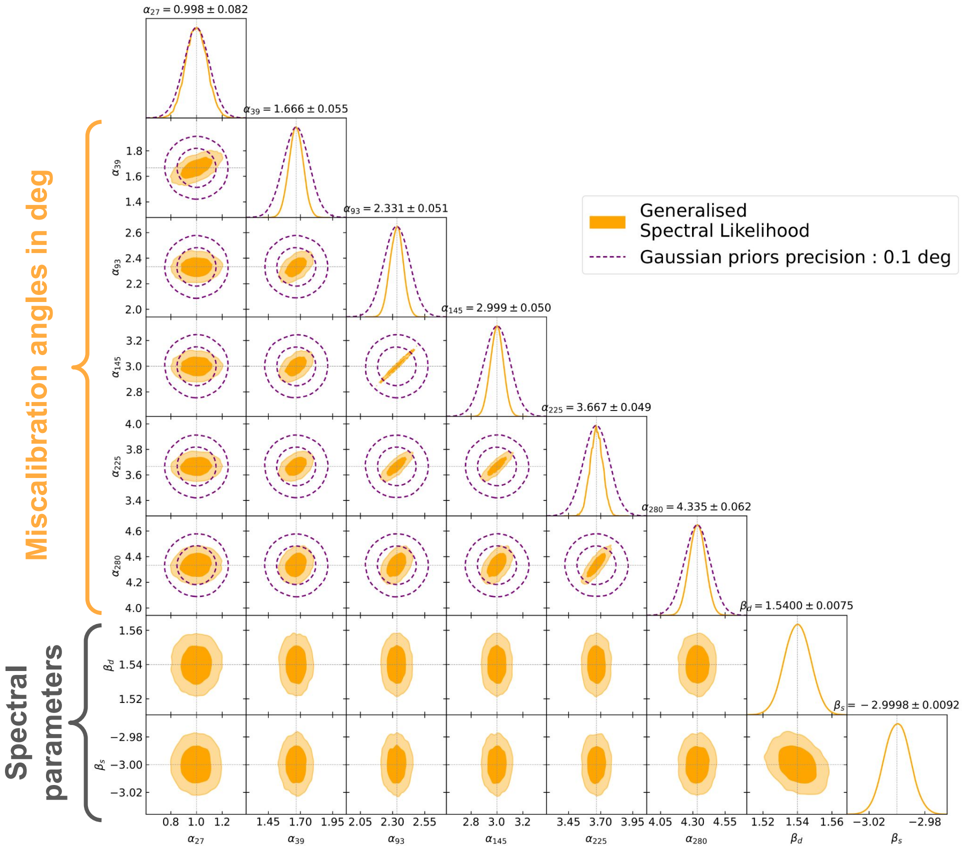
With only one prior of $\sigma(\alpha_{prior}) = 0.1°$ all polarization angle are retrieved with $\sigma(\alpha_i) \geq 0.1°$



Multiple Priors

Adding priors improves the precision:

- 6 priors $\sigma(\alpha_{prior}) = 0.1°$
- $\sigma(\alpha_i) \geq 0.05°$



Jost et al. PRD 2023

Cosmological Parameters (1 prior)

For r:

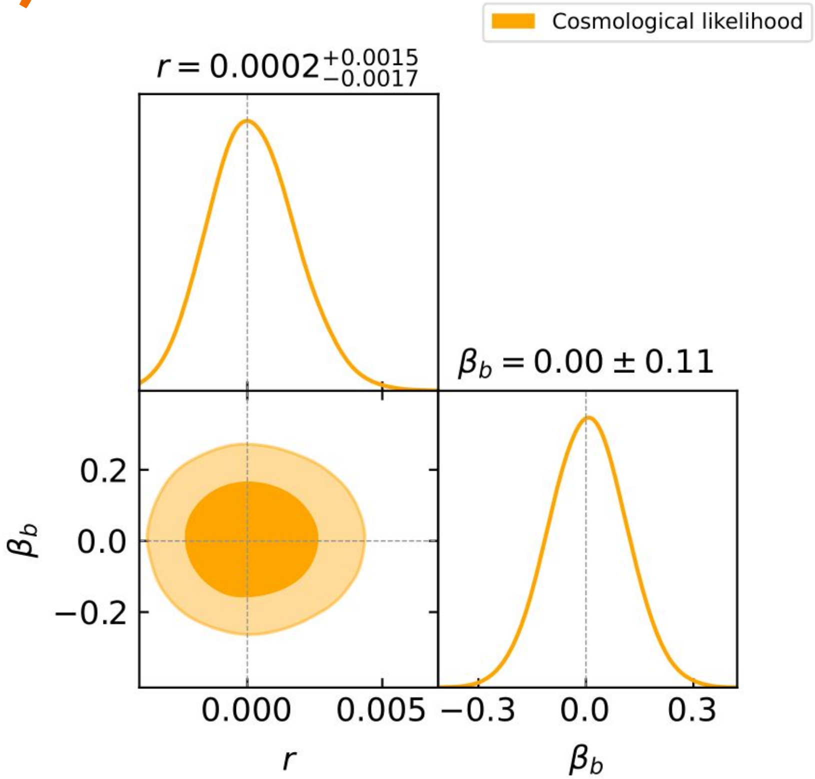
- Correct component separation
- Absolute angle marginalization $\boldsymbol{\beta_b} \Rightarrow$ no E→B leakage

⇒ no bias on r

For $\boldsymbol{\beta_b}$:

- Correct component separation
- Calibration priors

⇒ no bias on $\boldsymbol{\beta_b}$



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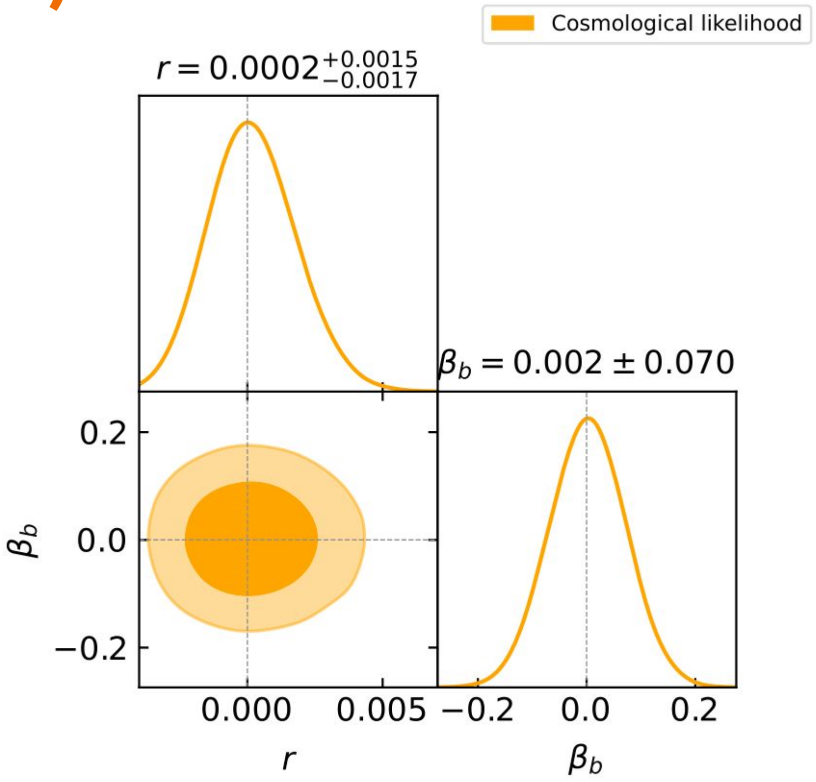
Cosmological Parameters (6 priors)

With 6 $\sigma(\alpha_{prior}) = 0.1°$ ⇒ $\boldsymbol{\sigma(\beta_b) = 0.07°}$

Enough for 5σ detection with current hints!



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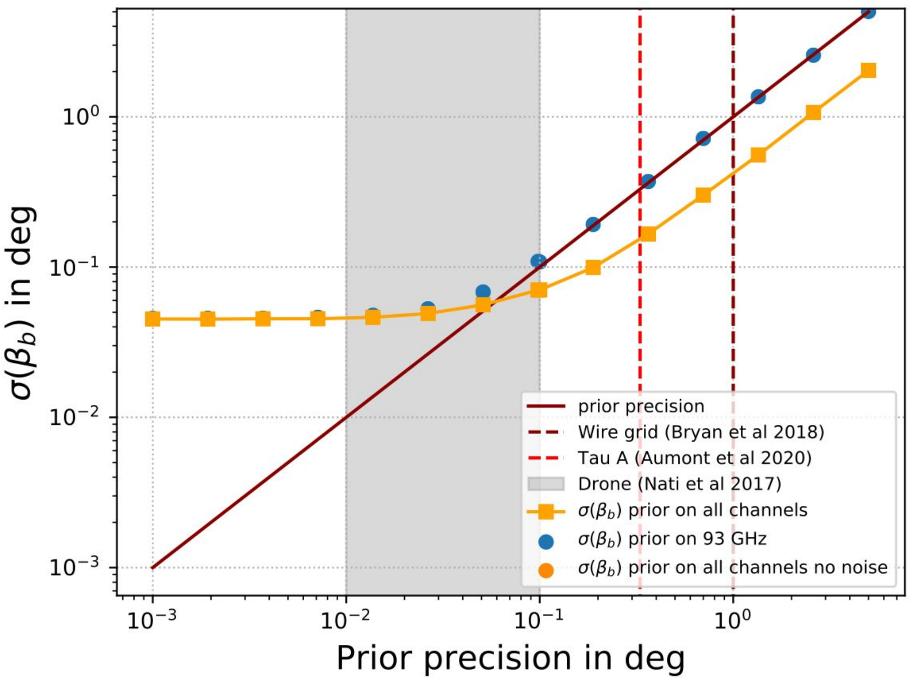
Evolution With Prior Precision

Input d0s0, with 1 or 6 priors

Two regimes:



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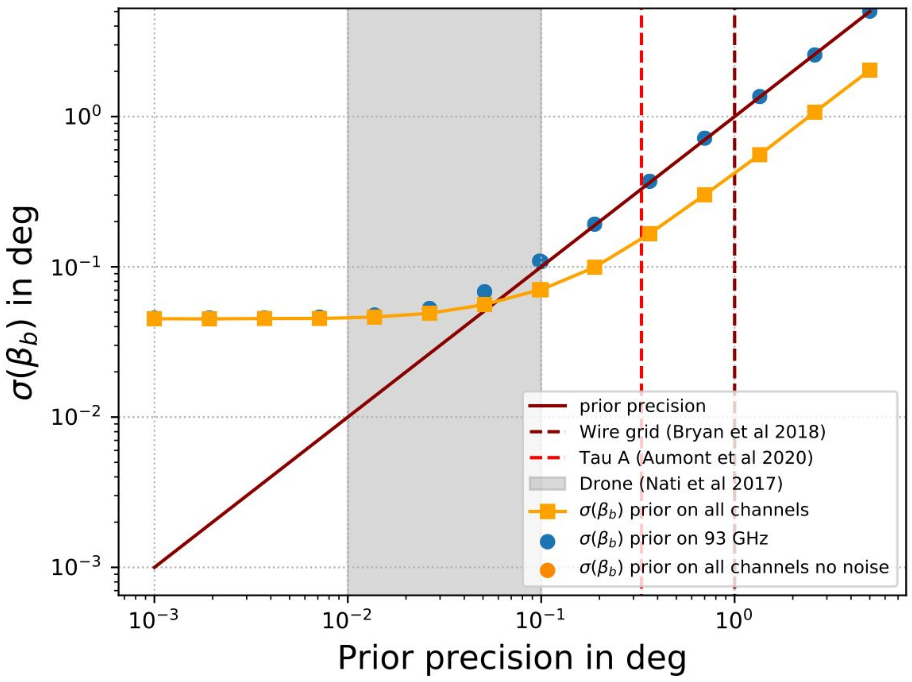
Evolution With Prior Precision

Input d0s0, with 1 or 6 priors

Two regimes:

- prior driven

Prior driven



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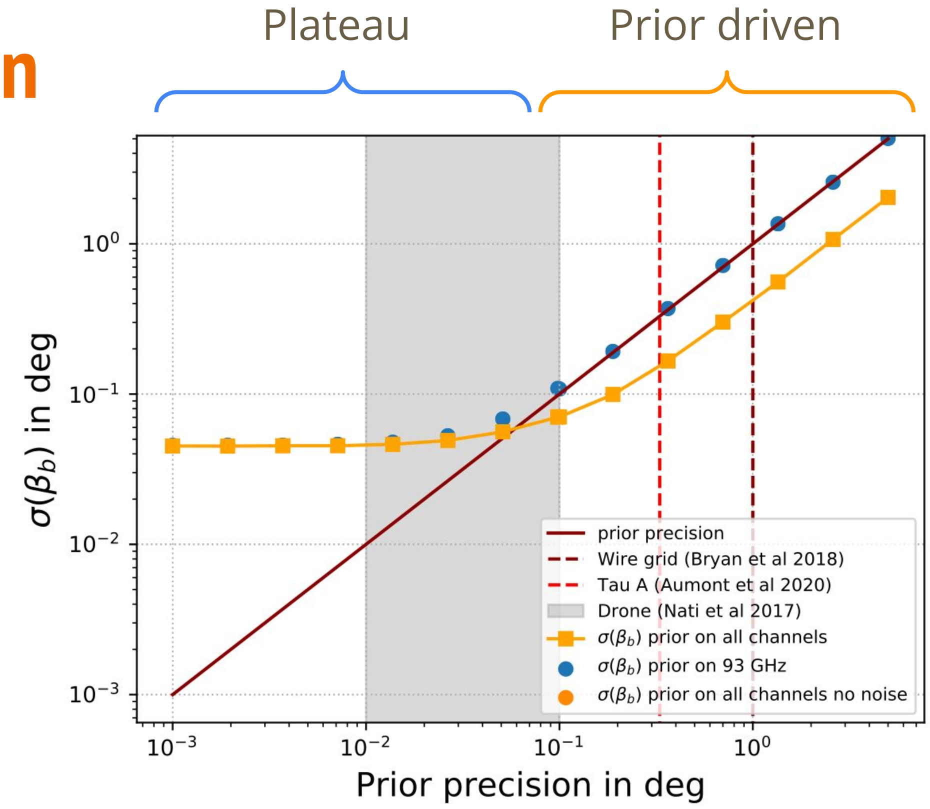
Evolution With Prior Precision

Input d0s0, with 1 or 6 priors

Two regimes:

- prior driven
- noise / cosmic variance plateau



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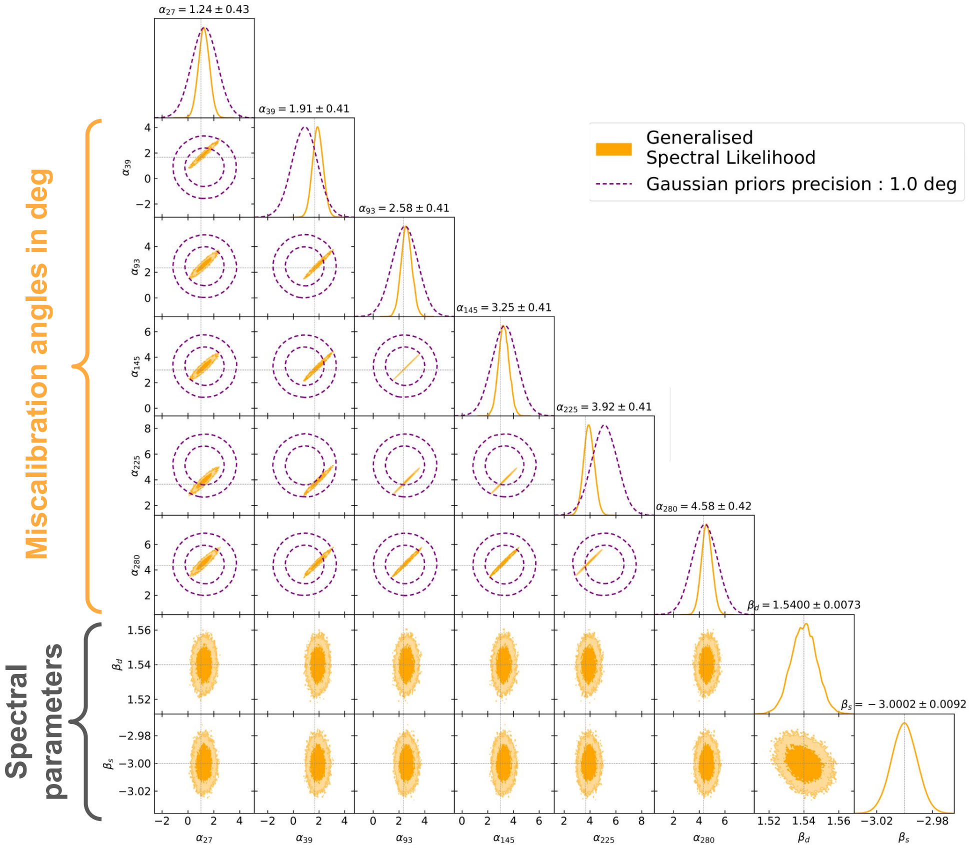
Biased Priors

With biased priors:

- Prior on all channels, $\sigma(\alpha_i) = 1°$
- Priors randomly biased by N(0,1°)
- All channels biased by the same amount (relative angles are still retrieved)



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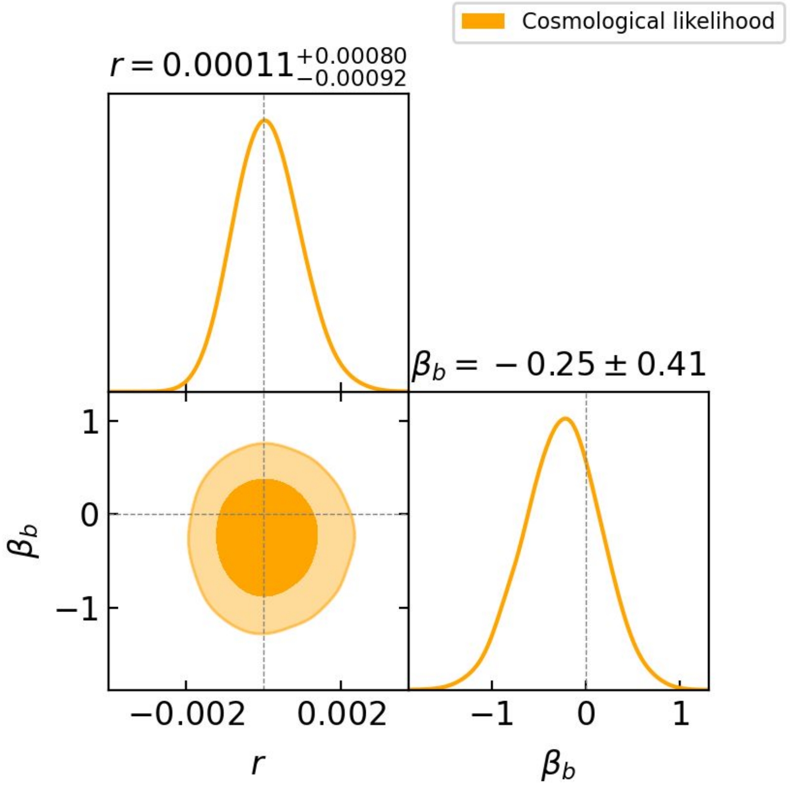
Biased Priors

With biased priors:

- **r** retrieved **without bias** (global angle marginalization)
- $\Delta(\beta_b) \approx \frac{1}{n_{\text{prior}}} \sum \Delta\alpha_{\text{prior}}$
- Having more independent priors should help reduce possible bias



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Conclusion

- Framework to include and optimise the information from calibration priors
- Can retrieve r and β_b
- Application to LiteBIRD forecast paper
- Input maps can come from different sources with different priors (or lack of) → open door for cross-calibration
- Future application to SO SAT analysis
- Need to include other effects currently working on filtering (work in progress in SO)

Thank you!